

¹ Exploring situations with real data for the probabilistic literacy of mathematics teachers

Explorando situaciones con datos reales para la alfabetización probabilística de profesores de matemáticas

Explorer des situations avec des données réelles pour l'alphabétisation probabiliste des professeurs de mathématiques

Explorando situações com dados reais para o letramento probabilístico de professores de matemática

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Abstract

In contemporary societies, people often face uncertainty and unpredictability, which justifies the need for probabilistic literacy. In the teaching of probability, the exclusive use of classical and decontextualized situations is commonly observed. This article aims to investigate how the use of real data can promote probabilistic literacy skills among pre-service and in-service mathematics teachers. It presents results from two qualitative studies conducted by members of the Research Group in Mathematics and Statistics Education (Gpeme): a doctoral thesis and a master's dissertation, both of which used real data to problematize the teaching of probability. The thesis involved five mathematics teachers from the Agreste region of Pernambuco, while the

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dissertation included seven undergraduate students from a federal public institution in the Sertão region of Pernambuco. In both studies, the goal was to present possibilities for learning probability to the participants. It was observed that the use of real data led to advances in the critical understanding of probability. Furthermore, when articulated alongside discussions of dispositional elements, such as beliefs and attitudes, it strengthened the participants' critical positioning. The study concludes that it contributes to mathematics education by proposing new perspectives for teaching probability, overcoming traditional approaches, and enabling new teaching practices focused on critical thinking and social justice.

Keywords: Probabilistic literacy, Real data, Teaching of probability, Pre-service and in-service teacher education.

Resumen

En las sociedades contemporáneas, las personas viven con frecuencia situaciones de incertidumbre e imprevisibilidad, y esta constatación se presenta como justificación para la alfabetización probabilística. En la enseñanza de la probabilidad, se observa generalmente el uso exclusivo de situaciones clásicas y descontextualizadas. Este artículo busca investigar cómo el uso de datos reales puede promover habilidades de alfabetización probabilística en profesores de matemáticas en formación inicial y continua. Se presentan resultados de dos investigaciones cualitativas desarrolladas por miembros del Grupo de Investigación en Educación Matemática y Estadística (Gpeme): una tesis doctoral y una disertación de maestría que utilizaron datos reales para problematizar la enseñanza de la probabilidad. Participaron en la tesis cinco profesores de matemáticas del Agreste pernambucano y en la disertación siete licenciandos de una institución pública federal del Sertão pernambucano. En ambas, se buscó presentar posibilidades de aprendizaje de probabilidad para los participantes. Se observó que el uso de datos reales permitió avanzar en la comprensión crítica de la probabilidad. Además, se identificó que, cuando se articula con discusiones relacionadas con los elementos disposicionales, como creencias y actitudes, se fortalece el posicionamiento crítico de los participantes. Se concluye que el trabajo contribuye a la educación matemática al proponer nuevas perspectivas para la enseñanza de la probabilidad, superando enfoques tradicionales y posibilitando que se vivan nuevas prácticas de enseñanza orientadas al pensamiento crítico y a la justicia social.

Palabras clave: Alfabetización probabilística, Datos reales, Enseñanza de la probabilidad, Formación inicial y continua.

Résumé

Dans les sociétés contemporaines, les personnes vivent souvent des situations d'incertitude et d'imprévisibilité, et ce constat sert de justification à la littératie probabiliste. Dans l'enseignement de la probabilité, on observe généralement le recours exclusif à des situations classiques et décontextualisées. Cet article vise à étudier comment l'utilisation de données réelles peut promouvoir les compétences en littératie probabiliste chez les enseignants de mathématiques en formation initiale et continue. Il présente les résultats de deux recherches qualitatives menées par des membres du Groupe de recherche en éducation mathématique et statistique (Gpeme) : une thèse de doctorat et un mémoire de master qui ont utilisé des données réelles pour problématiser l'enseignement de la probabilité. Cinq enseignants de mathématiques de la région d'Agreste dans le Pernambouc ont participé à la thèse, et sept étudiants en licence d'une institution publique fédérale de la région du Sertão dans le Pernambouc ont participé au mémoire. Dans les deux études, l'objectif était de présenter aux participants des possibilités d'apprentissage de la probabilité. Il a été observé que l'utilisation de données réelles a permis de progresser dans la compréhension critique de la probabilité. De plus, lorsqu'elle est articulée avec des discussions liées aux éléments dispositionnels, tels que les croyances et les attitudes, elle a renforcé le positionnement critique des participants. L'étude conclut qu'elle contribue à l'éducation mathématique en proposant de nouvelles perspectives pour l'enseignement de la probabilité, en dépassant les approches traditionnelles et en permettant de vivre de nouvelles pratiques d'enseignement axées sur la pensée critique et la justice sociale.

Mots-clés : Littératie probabiliste, Données réelles, Enseignement de la probabilité, Formation initiale et continue.

Resumo

Nas sociedades contemporâneas, as pessoas vivenciam frequentemente situações de incerteza e imprevisibilidade, o que justifica o letramento probabilístico. No ensino de probabilidade, observa-se geralmente a utilização exclusiva de situações clássicas e descontextualizadas. Busca-se neste artigo investigar como o uso de dados reais pode promover habilidades de letramento probabilístico em professores de matemática em formação inicial e continuada. Apresentam-se resultados de duas pesquisas qualitativas desenvolvidas por membros do Grupo de Pesquisa em Educação Matemática e Estatística (Gpeme): uma tese e uma dissertação que utilizaram dados reais para problematizar o ensino de probabilidade. Participaram da tese cinco professores de

matemática do Agreste pernambucano e da dissertação sete licenciandos de uma instituição pública federal do Sertão pernambucano. Em ambas, buscou-se apresentar possibilidades de aprendizagem de probabilidade para os participantes. Observou-se que o uso de dados reais possibilitou avanço no entendimento crítico da probabilidade. Além disso, identificou-se que, quando articulado a discussões relacionadas aos elementos disposicionais, como crenças e atitudes, fortaleceu o posicionamento crítico dos participantes. Conclui-se que o trabalho contribui para a educação matemática ao propor novas perspectivas para o ensino de probabilidade, superando abordagens tradicionais e possibilitando que sejam vivenciadas novas práticas de ensino voltadas para o pensamento crítico e à justiça social.

Palavras-chave: Letramento probabilístico, Dados reais, Ensino de probabilidade, Formação inicial e continuada.

Explorando situações com dados reais para o letramento probabilístico de professores de matemática

Introduction

Reflections on the presence of probability in various everyday situations have been a prominent topic among researchers in probabilistic education (Batanero, 2005; Gal, 2005). In contemporary societies, people often experience uncertainty and unpredictability, and this is often cited as a justification for probabilistic literacy. Thus, in an attempt to offer probability learning possibilities, several countries have included topics in this area of knowledge in the school curriculum, including: Brazil (Lopes, 2008; Silva; Grando, 2023), Australia (Pfannkuch, Ben-Zvi, & Budgett, 2018; English, & Watson, 2016), Portugal (Fernandes, 2024) and the United Kingdom (Kapadia, 2009; Department for Education, 2021).

The National Common Curriculum Base (BNCC) (Brasil, 2018), in the Thematic Unit of Probability and Statistics, establishes the skills to be developed in basic education to explore the proposed objects of knowledge. However, in the teaching of probability, especially in the final years, the focus is on manipulating mathematical calculations or experiments, usually related to the classical meaning of probability discussed by Batanero (2005).

Textbooks tend to structure their discussions mostly from this perspective. Silva (2023) analyzed two of the ten works approved in the notice of the National Program of the Textbook (Programa Nacional do Livro e do Material Didático—PNLD), valid for the period from 2021 to 2024. In the analyses, it was identified that there is a prioritization of mathematical quantification and a lower index of contextualized situations that explore probabilistic concepts in real-world contexts.

Exclusive reliance on mathematical calculations in classical probability settings can reinforce the erroneous belief that events are deterministic (Ferreira, 2025). Thus, in addition to calculations, it is essential to develop skills to analyze real situations, assess risks, and make conscious decisions. These skills can be mobilized through contextualized lived experiences throughout the learning process (Mlodinow, 2009).

Thus, the need to reframe conceptions of probability and carefully reflect on their implications to enable people to make decisions is challenging. In this sense, we believe that mobilizing critical skills related to probability can help us overcome cognitive biases that lead to poor judgments and choices (Mlodinow, 2009).

In this context, the relevance of the probabilistic literacy model (Gal, 2005) stands out, as it can mobilize knowledge and dispositional elements to develop reading, writing, communication, interpretation, and decision-making skills for understanding different probabilistic concepts. The model seeks to broaden the understanding of probability and integrate it into everyday life (Eugenio, Monteiro, & Carvalho, 2022).

This article aims to investigate how using real data can promote probabilistic literacy skills among mathematics teachers in initial and continuing education. The results of two research studies developed by members of the Research Group on Mathematics and Statistics Education (Grupo de Pesquisa em Educação Matemática e Estatística - Gpeme) linked to the Postgraduate Program in Mathematics and Technological Education (Edumatec) of the Federal University of Pernambuco (UFPE) will be presented. These studies used real data to problematize the teaching of probability, from the perspective of probabilistic literacy (Gal, 2005).

The article is organized into four sections. The first presents the conceptualization of probabilistic literacy (Gal, 2005). In the second, some methodological elements and results of the research conducted with teachers in continuing education are described (Eugenio, 2019). In the third section, some methodological elements and results of the research conducted with teachers in initial education are also presented (Ferreira, 2025). Finally, the last section presents the final considerations, with notes from both studies on the use of real data in discussions of probability.

Probabilistic Literacy

Probabilistic literacy (Gal, 2005) can be conceptualized as a set of knowledge and attitudes necessary to deal with a wide range of real-world situations involving the interpretation or generation of probabilistic messages, as well as decision-making. The author argues that being “literate” in this area means not only mastering mathematical calculations but also having the skills to interpret, critically evaluate, and act in situations involving uncertainty and chance in everyday life, such as decisions related to health, finance, or news understanding.

Gal (2005) systematized a theoretical model composed of two types of interconnected elements: knowledge elements and dispositional elements. The subset of knowledge elements is classified into Big Ideas, Probability Calculus, Language, Context, and Critical Questions. In turn, the dispositional elements are organized into Critical Posture, Beliefs and Attitudes, and Personal Feelings regarding Uncertainty and Risk, as shown in Figure 1.

Figure 1

Probabilistic Literacy Model



Note. Based on Gal (2005).

The figure shows that probability is multifaceted and can be explored from different perspectives. Gal (2005) points out that exploring only one or two of these elements is not enough to develop probabilistic literacy skills; it is necessary to integrate work on these elements to promote effective learning.

In the set of knowledge elements, the Big Ideas address fundamental concepts for understanding probability in everyday life, such as randomness, independence, and variability.

The element called Calculus of Probabilities (Figuring Probabilities) refers to the knowledge, attitudes, and beliefs about the different ways of determining or estimating the probability of events, since it is essential that the citizen understands probabilistic messages from third parties or generates their own estimates. This element can be related to quantitative and qualitative methods for estimating the probability of events, such as the numerical probability scale (Aczel, 2007).

However, Gal (2005) states that this element is not limited to the domain of formulas and procedures, but covers, for example, the formal or informal understanding of different approaches to determine probability, as well as the integration of

information from different sources and the evaluation of evidence, since the information used to arrive at a number has different levels of quality and reliability.

Language addresses elements of communication in probabilistic statements, whether numerical or verbal, highlighting the importance of knowing how to express and interpret these messages. Context is another relevant element that presents ten different key areas in which probability can be explored, such as health, finance, business, and human behavior, among others. Finally, Critical Issues encourage us to reflect on the reliability of probabilistic information, considering factors such as the source, the meaning of the message, and the context in which it is presented.

In the subset of dispositional elements, Critical Posture refers to the willingness to question and reflect on probabilistic information that may be misleading or unclear. Beliefs and Attitudes involve opinions and behaviors influenced by personal experiences and cultural or social contexts.

Beliefs are psychologically sustained understandings of the world. For Philipp (2007, p. 259, our translation), "beliefs can be thought of as lenses that affect the view of some aspect of the world". On attitudes, Philipp (2007, p. 259, our translation) defines them as "a way of acting, feeling or thinking, which reveals someone's disposition or opinion [...] attitudes, as well as emotions, can involve positive or negative feelings".

Gal's (2005) probabilistic literacy model can be used to assist in learning the reading, writing, and communication skills necessary for interpreting or generating probabilistic statements. The elements that comprise its structure reinforce the relevance of studying probability for citizen education and for coping with the uncertainties encountered in people's daily lives.

Carvalho and Monteiro (2024) emphasize that the use of real data enables citizens to become agents of transformation of their reality beyond the school walls, with criticality, empathy and responsibility, working to improve the quality of life and social justice.

Carvalho and Monteiro (2024) emphasize that incorporating empirical data from social life into the educational process fosters the formation of subjects capable of intervening critically in their reality, transcending the limits of the school. Thus, from the perspective of these authors, real data can enable people to develop competencies in analysis, empathy, and social responsibility, guiding their actions to promote quality of life and strengthen social justice. In this article, we share aspects of two Gpeme surveys that included real data.

The studies presented in this article adopt a qualitative approach (Prodanov & Freitas, 2013). One of the research studies refers to Eugenio's doctoral thesis (2019), which involved teachers in continuing education settings. The other research relates to Ferreira's master's thesis (2025), which was conducted with pre-service mathematics teachers.

Exploring probabilistic literacy with final-year teachers

Eugenio's (2019) research aimed to analyze, within a group of mathematics teachers of the final years of elementary school (K-6 to K-8), competencies in teaching probability from a probabilistic literacy perspective. The data were produced through the participation of five mathematics teachers who worked with students in grades 6 to 9 at a private educational institution in the Agreste of Pernambuco.

Six continuing education meetings were held to discuss aspects of probability teaching from a probabilistic literacy perspective, to problematize the teaching of probability through teachers' interpretations of real data.

This article discusses the sixth formative meeting, entitled "Interpretation of real situations in the context of probabilistic literacy and evaluation of the continuing education process." In this meeting, we presented a real-world situation to address the probability that the Brazilian men's football team would reach the final of the 2018 World Cup (Figure 2).

Figure 2

Record of the news used by Eugenio (2019) in the formative meeting

Brasil tem 32% de chances de chegar à final da Copa, segundo pesquisa

Levantamento da FGV apontou que Seleção Brasileira é a que tem mais oportunidades de passar por todas as fases



(Alex Lives/Getty Images)



Daniela Barbosa

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Note. Barbosa (2018).

The news published by Barbosa (2018) reported that the 32% probability was the result of a study conducted by the School of Applied Mathematics at Fundação Getúlio Vargas (FGV EMap). The results indicated that the Brazilian team would have this percentage chance of playing in the final and, if it reached the final, a 21% chance of becoming a six-time champion.

It was evident that the Brazilian men's team had the most world titles and, given its winning past, the empirical probability suggested that Brazil's chance of being champion in 2018 was greater. However, we sought to mobilize teachers' attitudes so that they could demonstrate how they believed probabilistic news was produced, as well as their attitudes toward criticizing it.

Some of the questions proposed by the researcher were: Which teams went to the final? What was the winning selection? (France) Were there any errors in the calculation of probabilities? What could be related from this context to probabilistic literacy? The research situation challenged the participating teachers to reflect on the methodological criteria that supported the conclusion presented in the news headline.

It was questioned whether the calculations by Getúlio Vargas Foundation were based exclusively on the performance of the participating team or whether they also considered the results of the national teams in editions prior to the 2018 World Cup. By proposing this interpretive framework, we sought to prompt reflection among teachers on how probabilistic situations are analyzed in the media and to develop a critical posture.

Continuing, the professors answered the subsequent question: Which teams went to the final? What was the winning selection? (correct answer = France). Were there any errors in the calculation of probabilities? What can we bring from probabilistic literacy in this context? What did you think of this probabilistic context? This was one of the contexts that Gal (2005) identified as areas for improvement in the teaching of probability.

In analyzing contexts of sports competitions, it is essential to consider a range of variables beyond the immediate results of matches. Among these factors, we highlight the age group and height of the athletes, the degree of collective engagement, previous experience in common clubs, the specific conditions of the place of the dispute, including geographical aspects, such as altitude, in addition to elements of a psychological nature, such as the emotional state of the players, and of a physical nature, such as the level of preparation and conditioning. In addition, the influence of fans and other external factors cannot be overlooked, as these dimensions can affect the competitive outcome.

To present the results, a qualitative analysis of the data was conducted, including inductive analysis (Prodanov & Freitas, 2013). From this perspective, the interpretations of mathematics teachers in probabilistic situations, based on real data, are examined, as reported in an extract from Eugenio (2019). To preserve anonymity, the researcher was identified as PE, and the participating teachers as PF1–PF5. Below, we present the dialogues lived in the situation.

PE: The researcher asked the teachers to analyze the data from Brazil and France and asked which country was the champion (PE, 2019, n. p.).

Teachers: **France.**

PE: And who was the competitor in the finals? They played against Croatia... Which teams went to the finals? France and Croatia. (PE, 2019, n. p.).

PE: The researcher had the teachers analyze the chances of reaching the final and being champion: Brazil, Croatia, Belgium, and France. (PE, 2019, n. p.).

PF4: I think when you go to the GAME... everything changes, because there is the physical issue... the emotional issue... Do you understand? This is a set of factors that interfere a lot... The supporters... Until reaching the finals, there are "n" factors... People with yellow cards... The best player on the team may be injured... and so on! (PF4, 2019, n. p.).

PF1: There is another detail there... The elements used to produce the statistics were not the same at the time of the World Cup... Because the calculations were based on past results... other teams... other players, and an estimate was calculated... So the scenario of the thing changed... the Brazil team whose probability was calculated did not participate in the calculations that led to it... (PF1, 2019, n. p.).

PE: Does everyone realize it? Brazil was the best team until 2002. And it still is, in terms of titles... But PF1 said these calculations were based on teams prior to 2018, and this is very important data... We only have Italy and Germany, who are four-time world champions; the other teams have fewer titles... We had a winning track record... So let's look at all this history of great titles and players, and all this is taken into consideration when calculating those odds... (PE, 2019, n. p.).

PF1: Another detail is that the countries did not stop in time... they invested in technology... they studied Brazil itself as it played. And another very important thing... Can you imagine the impact of saying that Brazil would only have a 6% chance of reaching the final? What would this news cause? Who was going to watch the game, who was going to buy the ticket to go watch it? What would that be like? Which sponsor would invest in Brazil? PE: If this news were given by USP, UNICAMP, UNB, or UFPE's mathematics department, which are renowned universities here in Brazil... (PF1, 2019, n. p.).

PF1: What if it were G1? And it (G1) is not a reference... FGV is a reference... But did another source participate in the research? Understand? I would trust more in that octopus from Spain who said he would be champion (PF1, 2019, n. p.).

In the dialogue, it appears that the participant identified as PF1 conducted an analysis that extrapolated from the data explicitly presented in the question, thereby mobilizing elements of probabilistic literacy. This participant noted that the probabilistic calculations were elaborated by considering both information prior to the 2018 World Cup and matches that had already taken place in the ongoing edition, although the latter had less influence than the historical results of previous World Cups. In turn, PF4 noted that in sports competitions, outcomes are subject to constant change, as multiple factors can influence a team's victory or defeat.

Gal (2005) emphasizes the need for a critical posture towards probabilistic information. PF1 added that if the projections indicated negative performance by the Brazilian team, there would be significant repercussions across different sectors. Among the examples mentioned, we highlight the losses to the industry that produces the

national team's official shirts, the reduction in viewership on television stations due to the tournament being held in Russia, and the weakening of the population's patriotic engagement. In addition, such statistical results could directly affect the players' motivation, leading them to enter the field with the perception that their chances of victory are reduced.

The teachers' analysis, mediated by the researcher, shows that continuing education contributed to the development of a critical posture, stimulated by debates on problematizing issues. This movement favored a more consistent reading and aligned with the assumptions of statistical education, allowing probabilistic information to be examined in a reflective and reasoned way.

The researcher asked whether there was any probabilistic error in the analyzed situation, because the Brazilian team did not reach the finals, much less become champions.

PE: Were there any errors in the probability calculation? (PE, 2019, n. p.).

PF3: No (PF3, 2019, n. p.).

PE: How could we justify this to a person who came to ask us about the result? (PE, 2019, n. p.).

PF6: It's like PF1 said, right? The variables have changed... the psychological factor, Neymar's injury... The cold weather in Russia... the weather... The French team was young and tight-knit... around 60%... played together... There are several things that influence... (PF6, 2019, n. p.).

PF1: And probability works with uncertainty... They are the chances of something happening... She does not claim that this will happen... So it doesn't mean it is going to happen... **There is a probability; she makes a projection...** And for this projection to occur, several factors are involved... Who knew Dilma wouldn't be a senator? On the eve of the election, she was in first place... But nevertheless, it came in 4th. Whoever bet on that result as reliable got burned... Probability works with uncertainty. (PF1, 2019, n. p., **emphasis added**).

PF4: How likely is Brazil to go to the finals now in the Copa América? (PF4, 2019, n. p.).

The teachers confirmed that there were no mistakes in the probabilistic calculations but noted that multiple factors can influence a match's results. Among the examples mentioned, we highlight Neymar's injury, which could affect the team emotionally, as he was seen as one of the team's main players, and his absence would represent a significant technical loss. In addition, it was pointed out that, by its nature, probability deals with uncertainty and cannot fully predict the events in a situation.

What can be done is to calculate the probabilities of certain events, indicating which results are more or less likely to occur.

Apparently, the teachers demonstrated a good level of maturity in the analysis of probabilistic situations, which converges with the perspective presented by Lopes (2008) when discussing the rupture of the linearity paradigm, which historically privileges classical areas such as arithmetic, algebra, and geometry, relegating statistics and probability to a secondary role. This rupture tends to become more significant as teachers begin working with content focused on statistical education from a critical perspective, favoring the development of students' probabilistic literacy. In this context, continuing education is a fundamental strategy for promoting critical citizens capable of making decisions based on consistent, systematic analysis.

A pertinent element in the constitution of probabilistic literacy is the beliefs and attitudes teachers manifest, as evidenced throughout this article. In this context, one of the teachers explains why he stopped supporting the Brazilian selection, presenting his justification in the following excerpt.

PF1: I myself have a belief... I stopped rooting for Brazil... Why? Because earning what they earn... those fortunes... to do what they do... This is absurd... I think we are kind of a puppet... Earning loads of money... Even though they are good footballers, I am not a good footballer, but I do not cheer for the misfortune of the other... 1st They roll in dough in their clubs... They earn sponsorships... they earn a lot with advertising on side hustles... so there is an extra stimulus... Now, in this context (Copa America), I would like them to win to destroy the tale that Brazil needs a myth... A special person... No, they must be a team! (PF1, 2019, n. p.).

PF4: I also agree with you, friend. We must win without Neymar and end this story! (PF4, 2019, n. p.).

PF1: To say... we lost because Neymar was not... For me, Neymar is not the best player in Brazil... Because behind Neymar there is a machine... The mainstream media all over him! (PF1, 2019, n. p.).

PF5: He is the great overturner... he keeps falling, always falls... (PF5, 2019, n. p.).

PF1: Why is Neymar summoned? Because otherwise the sponsors comes down on them... It's a very big external pressure... There are other comrades who do not have the sponsorship he has and are not called... So we can understand the following: in this context, **this probability is used to maintain this fanaticism for BRAZIL and SELL A LOT...** This sport has not evolved anymore and is now just a bargaining chip! (PF1, 2019, n. p., **emphasis added**).

PF1: **For example, you have women's football...** that play with race... with grit... Because the money didn't go in there... You see male and female volleyball,

which are shocked there and do not have all this prominence... Because the buckets of money that go into football are something surreal... (PF1, 2019, n. p., **emphasis added**).

In the excerpt above, the teachers continued their analysis of the Brazilian team's performance in the FIFA Cup, highlighting that during the data collection period, the team was preparing to face the Argentine men's soccer team in another championship, the Copa América. In his reflections, the perception that the world of football has undergone significant transformations was evident, with economic factors exerting greater influence than aspects such as race, determination, and willpower, even allowing the manipulation of results.

In addition, the teachers stressed that the Brazilian team should demonstrate the ability to win even without Neymar, reinforcing that sports success does not depend solely on an athlete but on collective action guided by a common goal. PF1, in turn, declared that they no longer support the national team due to corruption in football. These analyses reveal constitutive elements of probabilistic literacy, since they articulate beliefs, attitudes, and critical interpretations about the factors that can influence sports results. In this sense, the probabilistic situation analyzed contributed significantly to advancing conceptual understanding and developing skills related to probabilistic literacy, as argued by Gal (2005).

Research conducted with teachers in continuing education showed that systematic work on probabilistic literacy fostered reflection on their beliefs and attitudes in the face of real probabilistic situations. Initially, teachers did not conceive of probability as a non-deterministic field capable of manifesting itself across social problems, such as finance, weather forecasting, investments, and health, among other contexts highlighted by Gal (2005).

At the end of the continuing education program, consisting of six meetings, the participants reported having developed a new vision and perspective on probability and its teaching. They recognized it as a legitimate field of mathematics and understood that, through the development of probabilistic literacy, it is possible to make more reasoned decisions and make assertive choices in the face of uncertainties present in everyday life.

Addressing probabilistic literacy with pre-service teachers from real data

Ferreira's (2025) research aimed to analyze the possible contributions of the problem situations approach using real data across key areas of probabilistic literacy in

the initial education of mathematics teachers. The data for this research were produced through three cooperative meetings, in which problem situations were explored using real data to problematize the teaching of probability.

Seven pre-service teachers from the degree course in mathematics from a federal public educational institution in the Sertão of Pernambuco participated in the research. The methodological actions were developed in three meetings held at the end of the first half of 2024, using motivational questions and five problem situations that employed real data to problematize the non-deterministic nature of probability and the understanding of probabilistic concepts.

The situations, based on real data, were designed to address key areas of probabilistic literacy (Gal, 2005). In this article, we discuss the results of the fourth problem presented to the pre-service teachers at the third meeting, which was assigned to the research and statistics area. This situation aimed to problematize the classical understanding of probability, which, in teaching, is usually restricted to calculating the ratio of favorable cases to possible cases to obtain a percentage.

The most common examples involve calculating the probability of getting heads or tails when flipping a coin, or of being born male or female. In these situations, there is a tendency to determine only the 50% value, without considering the meaning of this percentage in practice.

In the third meeting, the licensees experienced two moments. The first consisted of a coin-toss dynamic, carried out by each participant, to explore prior knowledge of the theoretical notion of a 50% probability of obtaining a coin toss. Subsequently, this knowledge was problematized through analysis of real data from hospital birth records in Pernambuco, examining the licensees' beliefs and attitudes in light of the data.

The data were obtained from the Integrated Health Surveillance Platform of the Ministry of Health (Ministério da Saúde, 2023), which provides information on child births in Brazilian cities. In the context of the research, data from two cities in Pernambuco, Trindade and Olinda, were used to problematize the classic 50/50 probability of male and female births, as detailed in Table 1.

Table 1

Problem situation 4: 50% chance: the probability of births in Pernambuco cities

Key area: Research and Statistics.

Note the birth records of babies in hospitals in two cities in Pernambuco (Trindade and Olinda) in the first four months of 2023, as recorded by the Integrated Health Surveillance Platform of the Ministry of Health.

Registration in Trindade Hospitals

Sex	Jan/20 23	Feb/20 23	Mar/20 23	Apr/20 23	Total
Female	17	14	25	1	57
Male	32	25	24	12	93
Total	49	39	49	13	150

Registration in Olinda hospitals

Sex	Jan/20 23	Feb/20 23	Mar/20 23	Apr/20 23	Total
Female	182	168	180	144	674
Male	178	162	184	149	673
Total	360	330	364	293	1347

Source: Ministry of Health (2023).

The odds of being born a boy or a girl is 50% for each. However, the Trindade hospitals recorded months in which more than 60% of the babies born were boys, while in the city of Olinda, the birth frequency was around 50% for both sexes.

- Did any of these results surprise you? Why?
- What conclusions do you draw from the data presented?
- Do you believe one sex is more likely to stand out over the other in smaller cities? Why?
- If you could predict the data of Trindade 10 years from now, what could happen to the probability of the birth of boys and girls?
- Do you believe that, in the long term, the relative frequency of births of boys and girls in Trindade will tend to approach 50%? Why?

Note. Ferreira's research data (2025).

After presenting the data, we emphasized to the licensees that in January, February, and April, more than 60% of children born in Trindade were male. From these results, some questions were raised to promote a critical discussion with the

undergraduates, encouraging them to reflect on possible factors that could explain the deviation from the expected theoretical probability of 50% in the events.

The proposed questions sought to identify pre-service teachers' understanding of the influence of sample size in the two cities, their beliefs about interpreting the context under discussion, how they would estimate the behavior of the number of long-term births, and their conceptual understanding of randomness and variation in real situations.

The results from the interventions in the research showed the relevance of addressing beliefs in probability teaching and of exploring the key areas and contexts for understanding this concept. In problem situation 4, the key area of research and statistics was explored to analyze whether beliefs and attitudes about probabilistic data would be evident among the licensees.

It is noteworthy that, in the initial dynamics (launch of the coins), the licensees expressed beliefs to justify results far from the expected 50%, such as the influence of the physical structure of the coin. However, we also found that, due to the pre-service teachers' cooperative involvement, they noted the sample size, indicating that increasing the number of entries could bring the results closer to the theoretical probability of 50%. That is, suggesting, even if implicitly, an understanding related to the law of large numbers and the frequentist meaning of probability (Batanero, 2005).

Regarding the problem situation, we found that, although the licensees have shown to re-signify the understanding of theoretical probability (in the context of dynamics), when they were confronted with real data, which problematized in practice the behavior of randomness and variation, they again faced difficulties in critically interpreting the behavior of probability.

Speech extracts from Ferreira's research (2025) are presented below, along with an inductive analysis of qualitative data (Prodanov & Freitas, 2013). The licensees were identified from *L1* to *L7*. When asked about what they interpreted from the results presented and about any surprise in view of the data, especially when considering the theoretical probability of 50% for males and females (Table 1), they responded as follows:

L2: **Surprisingly, it would be this April, right? Which was a big difference, because there were 12 males and only 1 female.** So, this big difference is scary, right? In view of this issue of probability. For example, if it were a question of which would be more likely, few people could give credibility to this data there (L2, 2024, n.p., **emphasis added**).

L6: I agree, because it was almost 100% in April, **almost only boys were born there** (L6, 2024, n.p., **emphasis added**).

L5: Yeah, looking at this data, you can see that the only month in which more women than men were born was in March. **In Trindade, it seems that only men are born; there must be very few women** (L5, 2024, n.p., **emphasis added**).

L2, L6, and L5 expressed distrust and doubt about the behavior of variability in the birth records, especially in the April data (higher records for male biological sex, 12 x 1). Another relevant aspect can be observed in L2's speech, where, when referring to the sequence of 13 births, L2 suggests that few people could lend credibility to the result.

We note again that beliefs act like lenses that generally distance critical and reflective positioning from the analysis of events. No attitudes were mobilized that could promote the reflection that, in the context in question, it is a sequence of equiprobable events, since any sequence of 13 births (MFMFFMMFMMFMF - 7M + 6F or FFFFFFFFFFFFFF - 13F) is equally likely to occur.

We sought to deepen the discussions a little more by questioning the licensees about their conclusions when comparing data from Trindade and Olinda, aiming to identify, in their speeches, their understanding of probabilistic concepts, especially the behavior of data variability in the two cities. L6 started the discussion, arguing about the differences in the data.

L6: So, in **Olinda, we see that it is within the perception that we have of 50%.** It's okay that it's not 50% for one and 50% for the other, but it's almost the same. In **Trindade, there is a greater difference between one and the other. So, I believe that the male sex is what dominates there in Trindade** (L6, 2024, n.p., **emphasis added**).

Researcher: Is one biological sex, whether male or female, more likely to stand out over the other in smaller cities?

L3: No, I understand that we are a little surprised because it varies, more or less, away from 50% [...], but **there is also the question, which I said at the beginning, right? Regarding the sample size, because there are two**

different cities and Trindade is small, so, increasing the sample size may bring it closer to 50% (L3, 2024, n.p., **emphasis added**).

L2: I agree, **we keep coming back to our beliefs, right? As much as we want to say, one of them really stands out in smaller cities**. We are evaluating according to these data and the pattern we are looking for here, **but in probability, in randomness, it is not exactly like that, right?** In my opinion, this is what happened because of these concepts that we have already been discussing (L2, 2024, n.p., **emphasis added**).

L6: That's right, this is random, because those who studied biology know why a boy or a girl is born. And this has nothing to do with the place. It also happens in families; **many women only have a male child, and this is very random. She will never know before an exam; it is very uncertain** (L6, 2024, n.p., **emphasis added**).

The initial considerations presented by L6 implicitly demonstrate an understanding of what is expected from the frequentist meaning of probability (Batanero, 2005); that is, that greater amounts of data tend to approach the theoretical probability, as observed in the city of Olinda. However, by suggesting that, in Trindade, the male sex would be dominant in births, this belief limits the critical interpretation of the event by considering only a specific slice of the data, which may not reflect the city's reality.

When asked whether one of the biological sexes would stand out in the number of births in smaller cities, L3 complemented the discussion, reinforcing that this was an incoherent idea and emphasizing the importance of considering the sample size, noting that Trindade would be a small city. She also said that the amount of data presented could justify the divergences observed. In addition, he noted that using a larger sample could yield different results, demonstrating an understanding of the frequentist meaning (Batanero, 2005).

We also noted that L2 began to demonstrate a reflective attitude towards L3's speech by recognizing that, once again, beliefs were being prioritized over critical understanding of probability. L2 mainly highlighted the role of randomness in the observed results, prompting L6 to reconsider its initial position, advocating the superiority of a single biological sex in the city of Trindade. L6 even mentioned, for example, other random situations, such as cases of women who had a sequence of male children and the uncertainty surrounding these events.

Two other final questions were explored to identify how pre-service teachers would predict the data on Trindade in the coming years and whether, in the long term, these data would tend to approach the expected theoretical probability. This time, it was possible to observe that the licensees began to adopt a critical posture toward the results, as highlighted in the following excerpts.

L6: It can get closer, yes; **it can happen that, in the following months, more women are born too and reverse the situation**, because, as L2 said, they are random things (L6, 2024, n.p., **emphasis added**).

L5: It's like L3 said, there **would be more data, right? Here, we have only until April, and as it is random, it may be until in the other months even more women were born**; we do not know, we would have to check the other months better, **so I think it may approach 50%, because if this happened in March, then it may happen again** (L5, 2024, n.p., **emphasis added**).

L2: In the long term and with more data, **Trindade will be the same as Olinda, with a very close number of boys and girls** (L2, 2024, n.p., **emphasis added**).

Apparently, the licensees were more familiar with the frequentist meaning of probability, agreeing that, in the long term, Trinity records could approach the 50% predicted by theoretical probability. We also sought to highlight a relevant point observed in L2's speech, which suggested an attention to the absolute results of these births. The researcher emphasized to the licensees that the frequentist meaning (Batanero, 2005) was related to the relative frequencies of events and not to the absolute results, as highlighted by Matthews (2017, p. 15): "If we want to understand the action of chance on events, we must focus not on each event, but on its relative frequencies".

In general, the data obtained in the problematization lived in problem situation 4 are considered to reinforce the relevance of mobilizing conceptual reflections in the teaching of probability through the use of real data. Initially, we observed that, in the context of the dynamics, the licensees seemed to demonstrate a better understanding of how the concepts were interpreted in the coin launch results. It is noteworthy that this was directly related to previous discussions experienced during the cooperative meetings of Ferreira's research (2025).

We found that when addressing another key area and context using real data, beliefs again influenced the interpretation of information. These results highlight the potential and need to integrate dispositional elements into the teaching of probability,

especially when addressing the big ideas, to strengthen conceptual critical understanding of probability, which is essential for interpreting probabilistic events in daily life (Ferreira, 2025).

Final considerations

This article aimed to investigate how the use of real data can promote probabilistic literacy skills among in-service and pre-service mathematics teachers in initial and continuing education, with discussions of two research studies developed by members of the Research Group on Mathematics and Statistics Education (Grupo de Pesquisa em Educação Matemática e Estatística—Gpeme) linked to the Postgraduate Program in Mathematics and Technological Education (Edumatec) of the Federal University of Pernambuco (UFPE).

The research conducted with teachers in continuing education (Eugenio, 2019) showed that the use of real data in the context of probabilistic news during the formative process played a fundamental role in developing a critical posture among teachers regarding the teaching of probability. Throughout the meetings, we observed that teachers began to adopt a more questioning attitude, recognizing the importance of encouraging students to perform critical analyses of probabilistic information in different contexts.

We also found that teachers recognized the need to incorporate probabilistic literacy into their pedagogical practices to prepare students to navigate the uncertainties inherent in the real world.

Another relevant aspect of the research concerns the evolution of the conceptual understanding of probability and of probabilistic literacy among mathematics teachers. Initially, in the first continuing education meetings, teachers presented only the concept of classical probability. However, through the researcher's mediation, teachers began to analyze various probabilistic situations and realized that the classical approach was insufficient to address the complexity of problems in contemporary society. Thus, they recognized that the classical probability corresponds to a specific case of equiprobability.

In a study conducted with pre-service teachers (Ferreira, 2025), the use of real data promoted critical reflection on the teaching and learning of probability during their

initial education. The proposed problematizations strengthened the citizen education of these pre-service teachers. By mobilizing beliefs, attitudes, and feelings in the face of uncertainty, licensees could recognize the relevance of probability not only as another object of mathematical knowledge but as an essential competence for making conscious decisions in different life situations.

There was also the understanding that probability refers to an estimate, not a deterministic result. Even when an event has a 50% theoretical probability of occurring, the variability of the results indicates other equally likely outcomes. For the pre-service teachers, this aspect reinforced the need to mature, in the teaching of probability, the idea that probabilistic calculations should not be interpreted as objective predictions of events, but that, by the law of large numbers, events are expected to approach their theoretical probabilities as the number of repetitions increases.

In both studies, the problematization of real data helped participants understand the non-deterministic nature of probability and develop a critical posture. These studies also made it possible to work on elements such as beliefs and attitudes, through which participants could re-signify previous understandings.

It is noteworthy that the research participants began to understand, in practice, how randomness and variability operate in probabilistic events, thereby strengthening their critical thinking. These actions enabled teachers in initial and continuing education to recognize the importance of promoting, in basic education, the teaching of probability aimed at the formation of the citizens' critical consciousness, thereby allowing them to develop skills for making conscious decisions.

The real situations discussed in this article, although they have been explored in the context of initial and continuing education, can be adapted for use at other levels of basic education and in other areas, as suggested by research with in-service teachers in continuing education and with pre-service teachers.

We believe the studies offer new perspectives on the teaching of probability, since, as discussed in the introduction, textbooks generally focus on classic, decontextualized situations (Silva, 2023). The situations used in both studies were constructed from a probabilistic literacy perspective and focused, especially, on

dispositional elements, starting with participants' beliefs and attitudes, to foster a more effective understanding of probabilistic concepts.

The development of situations with real data was a challenging process because, at various times, there was concern about their relevance and meaning for the research interventions. The probabilistic literacy process is complex but feasible through systematic work with the elements of knowledge and dispositional elements (Gal, 2005).

The research highlights the importance of working with real data to encourage greater citizen engagement and responsibility in their decision-making. This factor is essential in the teaching and learning process, as it contributes to the development of critical, reflective citizens committed to pursuing social justice.

We believed that, through effective teaching of probability and research, as presented in this article, it is possible to promote social change that will positively impact people's lives. For future studies, we highlight the importance of investigating how the teacher's classroom work aligns with probabilistic literacy and how students would respond to a new teaching format that challenges them to position themselves critically in the face of the information presented.

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