

COINCIDENCES AND KONSTATIERUNGEN. ASPECTS OF THE METHODOLOGICAL CREATIVITY IN SCHLICK'S SCIENCE-ORIENTED PHILOSOPHIZING

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ABSTRACT

Moritz Schlick (1882–1936) – the integrating figure of the Vienna Circle – is an inspiring thinker who philosophizes in the immediate vicinity of contemporary physics in particular and other empirical sciences including psychology as well as ethics. In the context of interpreting Einstein's (general) theory of relativity he wrote his "*Space and time in contemporary physics, an introduction to the theory of relativity and gravitation*" [*Raum und Zeit in der gegenwärtigen Physik: zur Einführung in das Verständnis der Relativitäts- und Gravitationstheorie*]" – first published in 1917. Schlick developed his conception of *space-time coincidences of events*. For the second edition he added the new chapter "X. Relations to Philosophy" using coincidences methodologically to connect terms which belong to different spaces of meaning. Starting in 1934 – in the context of the debate on *protocol sentences* mainly with Otto Neurath and Rudolf Carnap – he offered his approach of *Konstatierungen*² to answer the question: "What is to be regarded as our fundament of knowledge?" I will shortly discuss Schlick's term *coincidence*, move on to *Konstatierungen* and show some interrelations between them. I will argue for the methodological creativity in Schlick's science-oriented philosophizing by explicating the inner structure of *Konstatierungen* within my 2-dimensional language of analysis. Finally, I will compare Schlick's *Konstatierungen* with Kant's *synthetic a priori judgments* and Frege's *thoughts* as interrelated cases of two-dimensionally structured intermediate cases.

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² I will explain later why I decided to use the German expression throughout this paper and not one of the proposals to translate this term into English.

KEYWORDS

SCHLICK. COINCIDENCE. KONSTATIERUNG. KANT. FREGE.

1. PRELIMINARY REMARKS

There is an overarching agreement that Moritz Schlick (1882–1936)³ was the integrating figure of a circle⁴ which was initially called “Schlick Circle” by some members and is now usually subsumed under the term “Vienna Circle”. It is also well-known that he was a highly esteemed conversational partner not only by the members of his circle but also by a lot of other influential thinkers of his time. If it comes to the question regarding his persistent philosophical or theoretical achievements then the answer is not so clear. “Schlick’s important philosophical work has unfortunately not found the attention it deserves.” (CARNAP 1997, 20) Carnap himself offers a whole list of the “later, often more elaborate and formalized developments by other authors”.⁵ (ibid) The elements of his list are

- his conception of the task of philosophy as an analysis of the foundations of knowledge and, in particular, of science, in other words, a clarification of meaning;
- the conception of meaning as given by the rules of the language for the use of a sign;
- the view that knowledge is characterized by symbolization and is thus fundamentally different from mere experience;
- the emphasis on the procedure, suggested by Hilbert's formalistic method, of introducing concepts by so-called implicit definitions, i.e., by postulates;
- the conception of truth as consisting in the unique co-ordination of a statement to a fact;

³ Friedrich Albert Moritz Schlick: 14.04.1882 (Berlin) – 22.06.1936 (Vienna).

⁴ “The congenial atmosphere in the Circle meetings was due above all to Schlick’s personality, his unfailing kindness, tolerance, and modesty. Both by his personal inclination toward clarity and by his training in physics, he was thoroughly imbued with the scientific way of thinking.” (CARNAP, 1997, 20) It is characteristic for Carnap that he uses “the scientific way of thinking” and not “the philosophical way”!

⁵ It is for sure that Carnap himself is one of the most important thinkers falling under “other authors.”

- the view that the distinction between the physical and the mental is not a distinction between two kinds of entities, but merely a difference of two languages;
- the rejection of the alleged incompatibility of freedom of the will and determinism as based on a confusion of regularity with compulsion. (CARNAP, 1997, p. 20)

It is interesting that Carnap mentions “the foundation of knowledge”. It is the English version of the title of Schlick’s article (1934) where he introduces his controversially discussed term *Konstatierung* which is a *philosophical* alternative to Carnap’s own *theory-oriented* proposal: *protocol sentences*. We observe that the list does not contain the expression *coincidence*.

In the context of interpreting Einstein’s (general) theory of relativity he wrote his „*Space and time in contemporary physics, an introduction to the theory of relativity and gravitation*“ [*„Raum und Zeit in der gegenwärtigen Physik: zur Einführung in das Verständnis der Relativitäts- und Gravitationstheorie“*]⁶—first published in 1917—Schlick developed his conception of *space-time coincidences of events* [*Raum-Zeit-Koinzidenzen von Ereignissen*]. For the second edition (SCHLICK, 1919) he added the new chapter “X. Relations to Philosophy” considering *pointing-coincidences* and *sensation-coincidences*. The reader will find further extensions in the third (SCHLICK, 1920) and the final version in the fourth edition (SCHLICK, 1922).

Starting in 1934—in the context of the debate on the so-called *protocol sentences* mainly with Otto Neurath and Rudolf Carnap—he offered his approach of *Konstatierungen* to answer the question: “What is to be regarded as our fundament of knowledge?” My fascination with this new term goes back to MAX (1998). Schlick’s article was a real zinger. Carnap immediately reacts: “My main concerns on your views are focused ... on the strange character of the ‘statements’ [*Konstatierungen*]. It is especially unclear to me how the derivation of such sentences, in which ‘here’ and ‘now’ occurs, and to which essentially indicative gestures belong, from a hypothesis to be checked should look like; this derivation

⁶ Throughout this text I put the original German expressions into square brackets.

is certainly necessary for the verification.”⁷ Already in 1935 the French translation « Sur le fondement de la connaissance » (SCHLICK, 1935fr-B/1934) was published together with some other papers on the same topic first published in French (SCHLICK, 1935fr-A, -C and -D). English translations were published much later (SCHLICK, 1956engl/1934; SCHLICK, 1979engl/1934 1979engl-a and -b).

There is a significant amount of literature about *coincidences* and *Konstatierungen* separately. We can distinguish several perspectives to look at the work of Schlick with respect of the two terms. We have a lot of *diachronic* studies motivated by answering the question concerning the (dis-)continuity in Schlick’s thinking (cf., i.e., FRIEDL, 2013; NEUBER, 2012; RYCKMAN, 1991, introductions and comments in SCHLICK, 2006; 2008). Because Schlick’s estate will be published in the *Moritz Schlick Gesamtausgabe* until 2030 we have more and more material at hand.⁸

There are a lot of very detailed *synchronic* investigations. In the case of (space-time) *coincidences* they are focused the discussions on the interpretation and understanding of the theories of relativity between Schlick and Einstein (CHAPUIS-SCHMITZ, 2005; ENGLER, 2009; FERRARI, 2005; GLASSNER, 2009, RYCKMAN, 1992; 2005) as well as Reichenbach (ENGLER, 2008). In the case of *Konstatierungen* we have two important contexts: (a) the protocol sentence debate, mainly with Carnap and Neurath (OBERDAN, 1996; 1998; 1999, UEBEL, 1996; 1999; 2007) and (b) Wittgenstein’s own philosophical developments (UEBEL 2020, MAX (*forthcoming*)). There is one proposal to look at *Konstatierungen* from a linguistic point of view (MAHLER, 2017).

⁷ My translation of: „Meine Hauptbedenken gegen Deine Auffassungen richten sich ... auf den seltsamen Charakter der ‚Konstatierungen‘. Besonders ist mir unklar, wie die Ableitung solcher Sätze, in denen ‚hier‘ und ‚jetzt‘ vorkommt, und zu denen wesentlich hinweisende Gebärden gehören, aus einer nachzuprüfenden Hypothese aussehen soll; diese Ableitung ist ja für die Nachprüfung sicherlich erforderlich.“ Letter from Carnap to Schlick on May 17th, 1934. You can find other skeptical remarks in HEMPEL, 1935 and later in DAVIDSON, 1987.

⁸ Cf. the review of three volumes of the *Moritz Schlick Gesamtausgabe* (MAX et al. (*forthcoming*)).

This paper is not discussing the pros and cons of the positions of the mentioned authors.⁹ Analyzing selected parts of Schlick's work and using my own language of analysis I will pay attention to the *inner structures* of both terms. But by showing some interesting correlations between *coincidences* and *Konstatierungen* this paper is a contribution to the discussion with respect to the (dis-)continuity in Schlick's philosophizing.

I am convinced that Schlick's investigations can be used very fruitfully to create inspiring links between pure theory and empirical data, between rational and factual truths, between formal logic and its applications, between rationalist and empiricist. Schlick demonstrates a method of inventing complex terms with an interesting *inner* structure which can be very helpful to discuss philosophical problems. "A philosophical problem has the form: 'I don't know my way about.'" (WITTGENSTEIN, 1986, p. 49: PI 123)¹⁰ Schlick is not interested in showing a solution by providing a detailed theory. But – unlike Wittgenstein – he is also not trying to show the borderlines between philosophy and scientific theories by presenting interrelated examples. One of his *methods* is to show how apparent gaps between concepts can be bridged. Finally he is mainly interested in the points of contact between science and data as well as between science and philosophy from his own philosophical point of view.

2. SCHLICK'S COINCIDENCES AND THEIR DIMENSIONALITY

There is a manuscript¹¹ in Schlick's estate from 1910 called "The limits of scientific concept formation" („Die Grenzen der naturwissenschaftlichen Begriffsbildung") in which we can find: "The psychic phenomena are not spatial, but it is well known that spatial relationships offer us the only possibility of measuring. All measurements take place in such a way that spatial coincidences

⁹ Using the results of this paper the critical discussion of other positions should be a subject of subsequent papers on these topics.

¹⁰ „Ein philosophisches Problem hat die Form: »Ich kenne mich nicht aus.«" (WITTGENSTEIN 1984, p. 302: PU 123).

¹¹ The entry in Fabian (2007, 15) reads: "151/A.97-1 Die Grenzen der naturwissenschaftlichen Begriffsbildung. Undat., p. 1-23, Handschrift / A5/. (A.97-1 bis A.97-7 bilden sieben aufeinander folgende Teile eines Heftes.)".

(galvanometer deflections, clock position, etc.) are observed.”¹² The context here is Einstein’s special theory of relativity (1905). We observe that Schlick is at the beginning only interested in measuring and observations of *spatial coincidences*. He is not mentioning *time*. We can call coincidences which “occur” in the same dimension *1-dimensional*.

He keeps this restriction in “The philosophical foundations of mathematics” [*„Die philosophischen Grundlagen der Mathematik“*]:

This also shows that the concept of equality only depends on what is retained [invariant] in this distorted world view: the respective coincidence of the body points.¹³

What touched before is touching now. Which spatial differences I can perceive in the world can only be described with the help of such coincidences of points of objects. The general theory of relativity makes use of this. It only has these, and also assumes that other laws apply in smaller districts. The perception does play a small role: the real geometry is chosen so that the immediate impression is not disturbed. But it is the ‘empirical’ perception, not the ‘pure’ one. The proof of the relativity of space shows that we have to understand equality of space as a physical concept.¹⁴

In *Space and time in contemporary physics, an introduction to the theory of relativity and gravitation* Schlick analyzes “the space-time-coincidence of two or more points”.

It is easily seen that the possibility of observing accurately depends upon noting identically the same physical points at various times and in various places; and that all measuring reduces itself to establishing that two such points, upon which we have fixed, coincide at the same place and at the same

¹² My translation of: „Die psychischen Phänomene sind unräumlich, uns bieten aber bekanntlich räumliche Verhältnisse die einzige Möglichkeit des Messens. Alle Messungen finden in der Weise statt, dass räumliche Coincidenzen (Galvanometeraussschläge, Uhrzeigerstellung etc.) beobachtet werden.“ 151/A.97-1, 9 right. It is remarkable that we cannot find this passage in the published version (SCHLICK, 1910).

¹³ My translation of: „Auch dies zeigt, dass es bei Begriff der Gleichheit bloss auf das ankommt, was in diesem verzerrten Weltbild erhalten bleibt: Die jeweilige Koinzidenz der Körperpunkte.“ (SCHLICK, 2006, p. 169).

¹⁴ My translation of: „Was sich vorher berührte, das berührt sich auch jetzt. Was ich in der Welt an räumlichen Unterschieden wahrnehmen kann, ist nur zu beschreiben mit Hilfe solcher Koinzidenzen von Punkten der Gegenstände. Davon macht die allgemeine Relativitätstheorie Anwendung. Bei ihr kommen nur solche vor, nimmt ausserdem noch an, dass in kleineren Bezirken noch anderen Gesetzmässigkeiten gelten. Eine kleine Rolle spielt die Anschauung doch: die wirkliche Geometrie ist so gewählt, dass der unmittelbare Eindruck nicht gestört wird. Aber es ist die ‚empirische‘ Anschauung, nicht die ‚reine‘. Der Beweis der Relativität des Raumes zeigt, dass wir Gleichheit des Raumes als physischen Begriff auffassen müssen.“ (SCHLICK, 2006, p. 170).

time. A length is measured by applying a unit measure to a body, and observing the coincidence of its ends with definite points on the body. With our apparatus the measurement of all physical quantities resolves finally into the measurement of a length. The adjustment and reading of all measuring instruments of whatsoever variety – whether they be provided with pointers or scales, angular-diversions, water-levels, mercury columns, or any other means – are always accomplished by observing the space-time-coincidence of two or more points. This is also true above all of apparatus used to measure time, familiarly termed *clocks*. Such coincidences are therefore, strictly speaking, alone capable of being observed; and the whole of physics may be regarded as a quintessence of laws, according to which the occurrence of these space-time-coincidences takes place. Everything else in our world-picture which can *not* be reduced to such coincidences is devoid of physical objectivity, and may just as well be replaced by something else. [...] Consequently, all coincidences remain undisturbed by the deformation. (SCHLICK, 1920engl, p. 49 f.)

There are several remarkable points: (a) the moves from “spatial coincidences”, “coincidence of the body points” to “space-time-coincidence[s]”, (b) of “two or more points”, (c) “coincide” can mean – following Einstein – “infinitely near one another”. But it can also mean that two different determinations (space and time) yield one new determination with an *inner* structure. (d) Physical objectivity – the question about what there is – is directly connected with the reducibility to coincidences. (e) From a theoretical point of view *coincidences* have to be the invariant features of the (physical) world.

Later on it becomes clear that Schlick uses *coincidence* not only in the context of measurement and stability with respect to transformations but also in the contexts of *experiences, perceptual phenomena* (optical space, tactual space) and in *the act of creating* new basic (theoretical) terms:

Now it is important to understand quite clearly what particular experiences lead us to connect a perfectly definite element of optical space with a perfectly definite element of tactual space, and thereby to form the conception of a ‘point’ in objective space. For it is here that experiences arising out of coincidences come into account. (ibid, p. 81)¹⁵

¹⁵ „Es ist nun wichtig, sich klarzumachen, welche besonderen Erfahrungen dazu führen, ein ganz bestimmtes Element des optischen Raumes einem ganz bestimmten Element des haptischen zuzuordnen und dadurch den Begriff des „Punktes“ im objektiven Raume zu bilden. Es sind nämlich Erfahrungen über Koinzidenzen, die hier in Betracht kommen.“ (SCHLICK, 2006, p. 274)

Like in the case of space and time we have two dimensions of different character: *optical* space and *tactual* space. Both spaces have *elements*. A “point” in objective space is not a logical atom, i.e., an object without any *inner* structure, but a pair of two elements which belong to two different spaces, two different *dimensions*. To illustrate the inner structure of “points” in objective space let us take the following representation:

Let os_i with i any integer be a perfectly definite element of optical space with

$\{ \dots, os_{-2}, os_{-1}, os_0, os_{+1}, os_{+2}, \dots \}$ being our optical space.

Let ts_j with j any integer: be a perfectly definite element of *tactual* space with

$\{ \dots, ts_{-2}, ts_{-1}, ts_0, ts_{+1}, ts_{+2}, \dots \}$ being our tactual space.

Then $\{os_i, ts_j\}$ *could* be a “point” in objective space. But the objective space cannot be the set of all the pairs of the form $\{os_i, ts_j\}$. It is only the set of such elements where os_i and ts_j *coincide*, i.e., there must be some restrictions on the relation between i and j . Schlick gives some examples to illustrate the attractivity of the coincidence of elements which belong to different spaces:

In order to fix a point in space, we must in some way or other, directly or indirectly, *point* to it: we must make the point of a pair of compasses or a finger, or the intersection of cross-wires coincide with it (i.e. bring about a time-space coincidence of two elements which are usually apart). Now these coincidences always occur consistently for all the intuitional spaces of the various senses and for various individuals. It is just on an account of this that a ‘point’ is defined which is objective, i.e. independent of individual experiences and valid for all. An extended pair of compasses applied to the skin excites two sensations of pricking; but if I bring the two points together so that they occupy the same spot in optical space, I only get *one* sensation of pricking, and there is also coincidence in tactual space. (ibid, p. 81 f.)¹⁶

¹⁶ „Um einen Punkt im Raume festzulegen, muß man irgendwie direkt oder indirekt auf ihn hinzeigen, man muß eine Zirkelspitze oder den Finger oder ein Fadenkreuz mit ihm zur Deckung bringen, d. h. man stellt eine raum-zeitliche Koinzidenz zweier sonst getrennter Elemente her. Und nun stellt sich heraus, daß diese Koinzidenzen für alle anschaulichen Räume der verschiedenen Sinne und Individuen stets übereinstimmend auftreten: eben deshalb wird durch sie ein objektiver, d. h. von den Einzelerlebnissen unabhängiger, für sie alle gültiger „Punkt“ definiert. Ein geöffneter Zirkel ruft bei Applikation auf die Haut im allgemeinen zwei Stichempfindungen hervor; führe ich aber seine beiden Spitzen zusammen, so daß sie für den

Discussing the “Relations to Philosophy” Schlick puts the *action of pointing* into his approach. This will be also a component of determining the meaning of his *Konstatierungen*. By *pointing* or other types of *action* we are able to bring two components of different types together in order to get a “point” in the objective space. In his later approach he will use *indexical* expressions (“here” and “now”) in *Konstatierungen*.

Schlick tries to create a bridge between sense-data and theoretical terms. Coincidences in sensual space leads to theoretical constructions. The problem is that “coincidences which are expressed by the differential equations of physics are not immediately accessible to experience. They do not directly signify a coincidence of sense-data; they denote non-sensory magnitudes, such as electric and magnetic intensities of field and similar quantities.” (ibid, p. 83) Since electric forces are measurable they “can just as well signify elements of reality as colours and tones”. (ibid) But Schlick has to accept that there is no proof of the correctness of his position. In this context *coincidence* was used *methodologically* to argue for his philosophical conviction.

In the context of *coincidence* Schlick takes psychological experience into consideration. In a dispute with Cassirer he wrote:

In the first place he [Cassirer] sees it [the content of pure intuition] in the concept of the coincidence of the “world points” to which the general theory of relativity, as it is well known, traces all natural laws back. But I believe that precisely this “coincidence” cannot be understood as a mere epitome and node of a priori sentences, but it is initially a representative of a psychological experience of coincidence, just as the word “yellow” denotes a simple color experience that cannot be defined any longer. Only in this way it can play the mediating role assigned to it by theory between reality and scientific-conceptual construction. In other words: we have an empirical view before us.¹⁷

Gesichtssinn, im optischen Raume, denselben Ort einnehmen, so erhalte ich nunmehr auch nur *eine* Sticheempfindung, d. h. es besteht auch im Tastraum Koinzidenz.“ (SCHLICK, 2006, p. 81 f.)

¹⁷ My translation of: „Cassirer sieht das Problem natürlich in seiner richtigen Bedeutung, und an zwei Orten seines Buches scheint er eine nähere Bestimmung des Inhaltes der vom logischen Idealismus behaupteten reinen Anschauung zu geben. An der ersten Stelle (S. 84) erblickt er ihn in dem Begriff der Koinzidenz der ‚Weltpunkte‘, auf welche die Allgemeine Relativitätstheorie bekanntlich alle Naturgesetze zurückführt. Aber ich glaube, daß gerade diese ‚Koinzidenz‘ sich garnicht als bloßer Inbegriff und Knotenpunkt apriorischer Sätze auffassen läßt, sondern zunächst durchaus Repräsentant eines psychologischen Erlebnisses des Zusammenfallens ist, so

When Reichenbach presumed that Schlick is, therefore, a psychologist, he answered:

If by it [to be a psychologist] (this would appear to me to be best historically justified) one understands the doctrine that the rules of logic are a kind of psychological law, then there is certainly no more wrong view, and I am certainly not a psychologist in that sense. But if one wants to characterize the assertion as psychologism, that none of our statements without exception would not be possible without certain psychological preconditions, then I don't see how one can escape this psychologism. It seems to me to be playing hide and seek from oneself when one, e.g., did not want to acknowledge that the sentences "there are constants in the world" and "there are experiences of equality" are for us after all equivalent. The logical is one ultimate, but the psychological is also one; one cannot be based on the other. (Schlick to Reichenbach, November 26, 1920)¹⁸

It is not possible to reduce the logical to the psychological and it is also not possible to reduce the psychological to the logical. But how can we bring them together? Schlick tries to show by using *coincidences* how we can integrate aspects which belong to different dimensions in one and the same picture of the reality: space-time, optical space, tactual space, "all the intuitional spaces of the various senses" and "a psychological experience of coincidence".

wie etwa das Wort ‚gelb‘ ein einfaches nicht mehr definierbares Farberlebnis bezeichnet. Nur so vermag sie die von der Theorie ihr zugewiesene Vermittlerrolle zwischen Realität und naturwissenschaftlich-begrifflicher Konstruktion zu spielen. Mit andern Worten: wir haben eine empirische Anschauung vor uns.“ (SCHLICK, 1920, p. 100 f.)

¹⁸ My translation of „Versteht man darunter [ein Psychologist zu sein] (dies würde mir historisch am besten gerechtfertigt erscheinen) die Lehre, daß die logischen Regeln eine Art psychologischer Gesetzmäßigkeit wären, so gibt es sicherlich keine verkehrtere Anschauung, und ich bin ganz gewiß nicht Psychologist in diesem Sinne. Will man aber auch schon die Behauptung als Psychologismus charakterisieren, daß alle unsere Feststellungen ohne Ausnahme nicht möglich wären ohne gewisse psychologische Voraussetzungen, so sehe ich nicht, wie man diesem Psychologismus entfliehen kann. Es heißt, scheint mir, vor sich selbst Verstecken spielen, wenn man z.B. nicht anerkennen wollte, daß die Sätze ‚es gibt Konstanten in der Welt‘ und ‚es gibt Erlebnisse der Gleichheit‘ für uns eben doch äquivalent sind. Das Logische ist ein Letztes, aber das Psychologische auch, eins läßt sich nicht auf das andere gründen.“

3. SCHLICK'S KONSTATIERUNGEN RECONSTRUCTED AS 2-Dimensionally Structured Intermediate Cases¹⁹

Schlick (1934) introduces the expression *Konstatierung* (plural: *Konstatierungen*) as an philosophical alternative to the term *protocol sentence* used by Carnap and Neurath since 1932. Because we have *constatation* in French the translator could use this term in Schlick (1935fr-A, -B, -C, -D). Unfortunately there is no such a corresponding term in English. Up to my mind the first English translation of “Über das Fundament der Erkenntnis” was published in Schlick (1959engl: “On the Foundation of Knowledge”). David Rynin saw the difficulty to translate “Konstatierung” very clearly:

The terms used by the author is “Konstatierung” which he sometimes equates with “observation statement” re. “Beobachtungssatz,” and generally lends to quote in a manner indicating his awareness that it is a somewhat unusual usage and perhaps a not altogether adequate technical term. Wilfred Sellars in a recently published essay (“Empiricism and the Philosophy of Mind,” Minnesota Studies in the Philosophy of Science Volume I University of Minnesota Press, 1956) uses the term “report” in referring to what seems to be the kind of statement Schlick is discussing. I do not adopt this term despite some undoubted advantages it has over “confirmation” because of the close connection that “Konstatierung” has with confirmation or verification, a connection so close that Schlick uses the same term unquoted to refer to confirmation. Furthermore, as the text shows, confirmations are never false as Schlick understands them but this is certainly not a characteristic of reports, as the term “report” is used in everyday or even scientific language. (Translator's note) (SCHLICK, 1959, p. 221 f.)

Peter Heath—who did the second English translation (SCHLICK, 1979engl)—choose “affirmation”. Let us summarize the situation in the following table:

¹⁹ In 2016 I presented my considerations—under the title “Schlick’s ‘Konstatierungen’ as structured intermediate cases between protocol statements and tautologies”—(a) at the IV Conference of the Brazilian Society for Analytic Philosophy University of Campinas (UNICAMP), July 5, 2016 - July 8, 2016, and (b) at PUC-SP, Grupo de pesquisa ‘Origens da Filosofia Contemporânea’, July 11, 2016.

| German | | English |
|-----------------|-------------|--|
| „Konstatierung“ | one to many | <p>“confirmation” (Schlick 1959engl)</p> <p>“affirmation” (Schlick 1979engl-b, Uebel 2020 etc.)</p> <p>“report” (Sellars 1956)</p> |
| German | | French |
| „Konstatierung“ | one to one | “Constatacion” |

Two problems arise: (1) If you search, i.e., for “affirmation” in English translations of Schlick’s text you will find a lot of occurrences which are not related to “Konstatierung” in the German source text. (2) All these translations have unwanted connotations which affect the interpretations by several authors in a problematic way.²⁰ To avoid unwanted connotations I will use the German term throughout this paper and add it in square brackets within English translations.

Our main text to be analyzed is of course “On the foundation of knowledge” (SCHLICK, 1979engl/1934). Let us start by having a closer look at some of Schlick’s examples for *Konstatierungen* (cf. SCHLICK, 1979engl, p. 385; 386):

- (a) “Here now blue.”
- (b) “Here now two black spots coincide [fallen zusammen].”²¹
- (c) “Here now blue is bounded by yellow.”
- (d) “Here now pain.”²²

²⁰ To show this in detail it needs a separate study.

²¹ Here we have a case in which the translator uses “coincide” for “fallen zusammen”. I.e., that we do not have “koinzidieren” in the German version. If we look only into the English text we get the impression that Schlick uses “coincide” to indicate a relation to his conception of *coincidence*. Maybe, there is such a connection. But we have to examine that.

²² These examples are taken from „Z. B. ‚Hier fallen jetzt zwei schwarze Punkte zusammen‘, oder ‚Hier grenzt jetzt gelb an blau‘, oder auch ‚Hier jetzt Schmerz . . .‘ usw.“ (Schlick, 2008, p. 511) and “Hier jetzt blau” (ibid, p. 512).

Schlick calls these constructions “statements” [*Aussagen*]. This is astonishing because (a) and (d) do not contain any verb. If he is using a (complex) verb (“coincide”, “is bounded by”) then their connotation is to space or time. Furthermore, he allows terms for colors (“blue”, “black”, “yellow”) and numbers (“two”).

“... these are always of the form ‘Here now so and so’.” (ibid, 385) The occurrences of “here” and “now” are necessary conditions for *Konstatierungen*. But they are not sufficient:

The mutually equivalent verbal forms:

‘Yellow here perhaps’

‘There seems to be yellow here’

‘Yellow here, if I’m not mistaken’

which, regarded as affirmations [*Konstatierungen*], would be meaningless, can now voice meaningful assertions. (ibid, p. 410)

I think the reason that these examples are not *Konstatierungen* is not the missing of “now”. The main point is that *Konstatierungen* cannot contain expressions like “perhaps”, “seems to be” and “if I’m not mistaken”. *Konstatierungen* cannot be doubted! Already the occurrence of the indexical “I” disqualify a statement of being a *Konstatierung*.

The throughout occurrences of “here” and “now” give us a first hint that there should be interesting interconnections with *space-time coincidences*. The demonstrative term “here” is indexically related to space and the demonstrative term “now” is indexically related to time. The fact that *Konstatierungen* have to contain both terms make it clear that we have a determination in *two different dimensions* once more: space and time. But the use of space-time coordinates in physics (e.g., represented by variables referring to points in space and time) with respect to *coincidences* is very different from the use of demonstrative term in *Konstatierungen*:

What is common to all these statements is that they contain *demonstrative* terms having the meaning of a present gesture, i.e., their rules of use stipulate that in making the statement in which they occur, an experience occurs, attention is directed to something observed. The meaning of the words

'here', 'now', 'this here', etc. cannot be stated by means of general definitions in words, but only through such words assisted by pointings and gesticulations. 'This here' makes sense only in combination with a gesture. In order, therefore, to understand the significance of such an observation statement, one must simultaneously make the gesture, one must in some way point to reality.²³ (SCHLICK, 1979engl, p. 385)

The utterance of constructions like "Here now blue." has to be supplemented by a context which shows several aspects: (i) It must have the meaning of a gesture. (ii) There are "rules of use" related to "an experience occurs" (e.g., to the sensation of *that* concrete color) and directed attention. (iii) Actions like "pointings and gesticulations" are needed. We could try to read the remark "cannot be stated by means of general definitions in words" as an indication that the *indexicals* could be used as undefinable basic expressions of some epistemology as *theory*. But then we are on the wrong track because pointings and gesticulations itself cannot be part of any theory! All this makes *Konstatierungen* very different from *coincidences*.

Nevertheless, there is another point which relates *Konstatierungen* and *coincidences* to each other if we look not only to *space-time* coincidences but also to coincidences concerning perceptions: the co-occurrence of space-time-related expressions and terms of perceptions like "blue" and "pain".

Let us proceed with the reconstruction of *Konstatierungen* as two-dimensionally structured intermediate cases within my language of analysis. The mostly relevant part of Schlick's text for our investigation is:

In other words, I can understand the meaning of an 'affirmation' ['Konstatierung'] only on and by way of a comparison with the facts, i.e., a carrying-out of the process required for the verification of all synthetic propositions. But whereas in all other synthetic statements [hypotheses], establishing the meaning and establishing the truth are separate, clearly distinguishable processes, in observation statements [Konstatierungen] they coincide, just as they do in analytic judgements. However different the

²³ „Das Gemeinsame aller dieser Aussagen ist, daß in ihnen *hinweisende* Worte vorkommen, die den Sinn einer gegenwärtigen Geste haben, d. h. die Regeln ihres Gebrauchs sehen vor, daß beim Aufstellen des Satzes, in dem sie vorkommen, eine Erfahrung gemacht, auf etwas Beobachtetes die Aufmerksamkeit gerichtet wird. Was die Worte ‚hier‘, ‚jetzt‘, ‚dies da‘ usw. bedeuten, läßt sich nicht durch allgemeine Definitionen in Worten, sondern nur durch eine solche mit Hilfe von Aufweisungen, Gesten angeben. ‚Dies da‘ hat nur Sinn in Verbindung mit einer Gebärde. Um also den Sinn eines solchen Beobachtungssatzes zu verstehen, muß man die Gebärde gleichzeitig ausführen, man muß irgendwie auf die Wirklichkeit hindeuten.“ (SCHLICK, 2008, p. 511 f.)

'affirmations' ['Konstatierungen'] may be from analytic propositions, they have this in common, that in both the process of understanding is at the same time the process of verification. Along with their meaning I simultaneously grasp their truth. To ask of an affirmation [Konstatierung] whether I might perhaps be mistaken about its truth, makes no more sense than with a tautology. Both have absolute validity. The analytical or tautological proposition, however, is at the same time devoid of content, whereas the observation statement gives us the satisfaction of a genuine acquaintance with reality.²⁴

We observe that Schlick uses expressions of three different but somehow interrelated types:

(a) "affirmations" [„Konstatierungen“], "observation sentences" ["Beobachtungssätze"]²⁵,

(b) "all other synthetic statements" [„alle anderen synthetischen Aussagen“], and

(c) "analytic propositions" [„analytische Urteile“], "analytical or tautological proposition" [„der analytische, der tautologische Satz“].²⁶

Without going into detail I give only an overview about some properties of these three types in the following table:

²⁴ „Mit anderen Worten: den Sinn einer ‚Konstatierung‘ kann ich nur dann und nur dadurch verstehen, daß ich sie mit den Tatsachen vergleiche, also jenen Prozeß ausführe, der bei allen synthetischen Sätzen für die Verifikation erforderlich ist. Während aber bei allen anderen synthetischen Aussagen die Feststellung des Sinnes und die Feststellung der Wahrheit getrennte, wohl unterscheidbare Prozesse sind, fallen sie bei den Beobachtungssätzen zusammen, ganz wie bei den analytischen Urteilen. So verschieden also auch die ‚Konstatierungen‘ von den analytischen Sätzen sind: gemeinsam ist ihnen, daß bei beiden der Vorgang des Verstehens zugleich der Vorgang der Verifikation ist; mit dem Sinn erfasse ich zugleich die Wahrheit. Bei einer Konstatierung hätte es ebensowenig Sinn zu fragen, ob ich mich vielleicht über ihre Wahrheit täuschen könne wie bei einer Tautologie. Beide gelten absolut. Nur ist der analytische, der tautologische Satz zugleich inhaltsleer, während der Beobachtungssatz uns die Befriedigung echter Wirklichkeitserkenntnis verschafft.“ (SCHLICK, 2008, p. 512)

²⁵ Schlick uses also „Aussagen über ‚gegenwärtig Wahrgenommens‘“ (SCHLICK, 2008, p. 505) ["assertions about the 'immediately perceived'"] (SCHLICK, 1979engl, p. 381)].

²⁶ It is very characteristic for Schlick's writing style that he uses not only one and the same expression or phrase with different meanings throughout his argumentation but several variations with the same meaning. We have to take the particular context into consideration in order to see this. If we, i.e., think that the meaning of „observation sentence“ is identical with the meaning of “hypothesis” we are on the wrong track. The same holds for thinking that the phrase “synthetic sentences” denotes exactly one type. Schlick divides “synthetic sentences” into “Konstatierungen” and “all other synthetic sentences”. This is in strong analogy with Kant: If we say “synthetic judgments” is remains open whether we mean “synthetic judgments a posteriori” or “synthetic judgments a priori”!

| | | |
|--|--|--|
| <i>hypotheses</i> "all other synthetic statements" | <i>Konstatierungen</i> special synthetic statements | <i>tautologies</i> |
| synthetic | synthetic | analytic |
| "establishing the meaning and establishing the truth are separate" | establishing the meaning and establishing the truth "coincide" | establishing the meaning and establishing the truth "coincide" |
| among them may be <i>Protocol sentences</i> | | <i>no observation sentences</i> |
| form: <i>a</i> [person] observed at <i>p</i> [place] at <i>t</i> [time] so-and-so | form: Here now so-and-so. | e.g.: equations like $3 + 1 = 4$ |
| "translation" changes the type!! | | |

Schlick has to fight against our prejudice that *Konstatierungen* seem to be closer related to *hypotheses* than to *tautologies*. Therefore, he mentions a lot of aspects (a) where *Konstatierungen* and *hypotheses* differ and (b) where *Konstatierungen* and *tautologies* coincide. Let us illustrate (a) by the following table:²⁷

²⁷ Cf. Schlick's argumentation in a letter to Ludovico Geymonat, January 2, 1936: „Sie glauben, ich hätte die Konstatierungen in dem Sinne als definitiv bezeichnet, dass durch sie eine endgültige Verifikation von Hypothesen stattfinde. Dies ist ganz offenbar ein Mis[s]verständnis, denn ich weiss natürlich – und der letzte Satz meiner Broschüre spricht es deutlich aus –, dass Hypothesen niemals endgültig verifizierbar seien. Ja, ich würde sogar noch weiter gehen als Sie (und z. B. Popper) und behaupten, dass sie nicht einmal endgültig falsifizierbar seien; im Prinzip nämlich könnte jede Falsifikation durch Einführung neuer Hilfhypothesen unwirksam gemacht werden. Das Wort „definitiv“ in seiner Anwendung auf die Konstatierungen bedeutet also etwas ganz anderes, nämlich dies, dass durch sie das Erlebnis einer erfüllten Erwartung, das Lustgefühl einer Bestätigung eintritt, falls der durch die Konstatierung ausgedrückte Tatbestand auf Grund irgendwelcher Hypothesen vorhergesagt war und erwartet wurde. Diese Bestätigung ist etwas Endgültiges, durch keine Korrektur Hinwegzudeutendes, ganz unabhängig davon, ob man nun daraufhin an die Richtigkeit der Hypothese glaubt (sie für verifiziert hält) oder nicht.“

| <i>hypotheses</i> | <i>Konstatierungen</i> |
|---------------------------------|---------------------------------------|
| stable | <i>not</i> stable |
| recordable | <i>not</i> recordable |
| <i>never</i> finally verifiable | finally verified (cannot be false) |
| rectifiable | <i>not</i> rectifiable |
| creations of reason | <i>no</i> creations of reason |

My argumentation with respect to (b) runs as follows: Schlick has to extend the reading of verification – usually reserved for synthetic sentences – to analytic sentences (tautologies, logical laws). I assume, that this step irritated Carnap and Neurath a lot. What should it mean to *verify a tautology*? It means to show that a sentence (a formula) is true in all possible situations (assignments, models). But does it mean to understand this tautology at the same moment? It means to see that this sentence (formula) is true in all possible situations. Take an example: To show that $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$ is a classical tautology we can *use* the method of (truth-)value-tables. We have to compute the values for each assignment of (truth-)values to propositional variables. But if this job is done we see *at the same moment* that this formula is a tautology. Incomplete showing is not seeing the whole picture which is necessary for *understanding*. In the case of *Konstatierungen* we have (verb-free) statements like “Now here blue”. It is like having exactly one assignment and this is sufficient for understanding as well as for verification at the same moment. In this case there is no alternative second assignment. Maybe Carnap could say – in the spirit of Frege – that an expression which has exactly one assignment cannot be a variable. It has to be a *constant* (maybe a propositional constant) naming the true. But this expression is logically equivalent to any other tautology. Carnap would doubt that there are *L-indeterminate* expressions representing empirical facts which have only one true assignment. And the formal objection would be that it should be possible to apply negation like in the case of a propositional constant. But *Konstatierungen* are not rectifiable. To say “It is not the case that here and now blue” makes no sense.

Our main point is, that this way of looking at Schlick's *Konstatierungen* is misguided. Because discussing the coincidence of establishing the meaning and establishing the truth is only one of their dimensions. We have to consider at least a second dimension: *Konstatierungen* are *synthetic* statements as well. This gives us already enough stuff to reconstruct *Konstatierungen* as two-dimensional structured intermediate cases. What are the two dimension? With respect to the first dimension (D1) we can divide statements into synthetic (D1A) and analytic (D1B) ones. Let us call the first dimension the *dimension of content*:

Dimension D1: *dimension of content*: [D1A] *synthetic* vs. [D1B] *analytic*.

D1A: If a statement is *synthetic* it expresses something about the objects of experience.

D1B: In *analytic* judgements "the question of their validity notoriously poses no problem. They are valid a priori, we must not and cannot be convinced of their correctness by experience, because they say nothing whatever about the objects of experience."²⁸ (SCHLICK, 1979engl, p. 383)

If we look at *Konstatierungen* only with respect to the first dimension then there would be no difference. One way out would be to introduce two ways of being synthetic, maybe being synthetic in some *strong* way and being synthetic in *weak* way. But this is not Schlick's strategy. He is convinced that *hypotheses* and *Konstatierungen* are synthetic in the same way. In order to get a difference we have to look at a second dimension (D2) with respect to the *relation* between understanding (D2) and verification (D2B). But to say that we *verify* hypotheses and *understand* *Konstatierungen* is not the solution which Schlick is going to offer us. Then we would be not able to bring tautologies into our play. Schlick offers here his most impressive creation. He introduces a *binary* relation between verification and understanding: "the process of understanding is at the same time the process of verification". I take the symbol " $=_t^{process}$ " to indicate that two (different) processes *coincide* with respect to the same moment *t* and the symbol

²⁸ „Bei analytischen Urteilen [D1B] spielt die Frage ihrer Geltung bekanntlich keine Rolle. Sie gelten a priori, man muß und kann sich von ihrer Richtigkeit nicht durch Erfahrung überzeugen, weil sie überhaupt nichts von Gegenständen der Erfahrung aussagen.“ (SCHLICK, 2008, p. 509)

$\neq_t^{process}$ to indicate that two (different) processes *does not coincide* with respect to the same moment t .

Dimension D2: dimension of understanding (D2A) / verification (D2B)

$D2 =_t^{process}$: understanding (D2A) $=_t^{process}$ verification [D2B]

I can indeed doubt whether I have correctly grasped the meaning of some sign-complex, and even whether I shall ever understand the meaning of a given word-series at all; but I cannot ask whether I am also really able to discern the correctness of an analytic proposition. For in an analytic judgement, to understand its meaning [D2A] and to discern its *a priori* validity [D2B], are *one and the same process* [$=_t^{process}$].²⁹ (ibid, p. 384 f.)

$D2 \neq_t^{process}$: understanding (D2A) $\neq_t^{process}$ verification [D2B]

A *hypothesis* (*synthetic* statement which is not a *Konfirmation*) “is characterized by the fact that if I have merely discerned its meaning [D2A], I have no notion whether it is true or false; its truth is established only by a comparison with experience [D2B]. The process of discerning the meaning is here entirely different [$\neq_t^{process}$] from that of verification.” (ibid, p. 385)³⁰

Now we are able to represent the *complete inner* structure of *hypotheses* and *analytic propositions* by using the following vertical structure in square brackets expressing the two dimensions together in one complex form:

The inner structure of *hypotheses* is represented by

$$\left[\begin{array}{c} \text{synthetic} \\ \text{understanding } \neq_t^{process} \text{ verification} \end{array} \right].$$

²⁹ „Wohl kann ich daran zweifeln, ob ich den Sinn irgendeines Zeichenkomplexes richtig erfaßt habe, ja ob ich überhaupt jemals den Sinn irgendeiner Wortreihe verstehen werde; aber ich kann nicht fragen, ob ich die Richtigkeit eines analytischen Satzes auch wirklich einzusehen vermag. Denn seinen Sinn verstehen [D2A] und seine apriorische Geltung einsehen [S2B], sind bei einem analytischen Urteil *ein und derselbe Prozeß*.“ (SCHLICK, 2008, p. 511)

³⁰ A hypothesis is „dadurch charakterisiert, daß ich durchaus nicht weiß, ob sie wahr oder falsch ist, wenn ich nur ihren Sinn eingesehen habe [D2A], sondern ihre Wahrheit wird erst durch den Vergleich mit der Erfahrung festgestellt [D2B]. Der Prozeß der Einsicht in den Sinn ist hier ein völlig anderer [$\neq_t^{process}$] als der Prozeß der Verifikation.“ (SCHLICK, 2008, p. 511)

The inner structure of *tautologies* is represented by

$$\left[\begin{array}{c} \text{analytic} \\ \text{understanding} =_t^{\text{process}} \text{verification} \end{array} \right].$$

They are different in both dimensions and, therefore, *completely different*. None of them can play the role of the fundament of knowledge with respect to the other one. If we accept that the fundament is not located at the bottom of the two and has no duration in time then we can look for an *intermediate case* by which we can connect the two types of statements.

If we turn our attention to the connection of science with reality, and see in the system of its propositions what it really is, namely a means of orienting oneself among the facts, of attaining to the joy of confirmation, the feeling of finality, then the problem of the 'foundation' will automatically transform itself into that of the unshakeable points of contact between knowledge and reality. (ibid, p. 386 f.)³¹

We already know the candidates for connecting science with reality by bridging the gap between *hypotheses* and *tautologies*: *Konstatierungen*:

The inner structure of *Konstatierungen* is represented by

$$\left[\begin{array}{c} \text{synthetic} \\ \text{understanding} =_t^{\text{process}} \text{verification} \end{array} \right].$$

Konstatierungen represented that way can be interpreted as *two-dimensionally structured intermediate cases* if we look at the following structure:

$$\begin{array}{ccc} \textit{hypothesis} & \textit{Konstatierungen} & \textit{tautologies} \\ \left[\begin{array}{c} \text{synthetic} \\ \text{und.} \neq_t^{\text{process}} \text{ver.} \end{array} \right] & = \left[\begin{array}{c} \text{synthetic} \\ \text{und.} =_t^{\text{process}} \text{ver.} \end{array} \right] & \neq \left[\begin{array}{c} \text{analytic} \\ \text{und.} =_t^{\text{process}} \text{ver.} \end{array} \right] \end{array}$$

These absolutely fixed points, the affirmations [*Konstatierungen*], we have come to know in their particularity; they are the only synthetic propositions which are not hypotheses. In no sense do they lie at the basis of science, but knowledge, as it were, flickers out to them, reaching each one for a moment only, and at once consuming it. And newly fed and strengthened, it then flares on toward the next. (ibid, p. 387)

³¹ „Richtet man das Augenmerk auf den Zusammenhang der Wissenschaft mit der Wirklichkeit, sieht man in dem System ihrer Sätze das, was es eigentlich ist, nämlich ein Mittel, sich in den Tatsachen zurechtzufinden, zur Bestätigungsfreude, zum Gefühl der Endgültigkeit zu gelangen, so wird sich das Problem des ‚Fundamentes‘ von selbst in das Problem der unerschütterlichen Berührungspunkte von Erkenntnis und Wirklichkeit verwandeln.“ (SCHLICK, 2008, p. 514)

Konstatierungen are located *within* science and do not lie on the bottom. That's not what we normally would expect of a "Fundament"! It remains open in which way the two dimensions within a *Konstatierung* has to be interrelated in order to make plausible that each *Konstatierung* exists "for a moment only". But this aspect is also questionable with respect to *coincidences*.

We take schema above and go back to our "mostly relevant part" of Schlick's text and supplement it with the relevant parts of our structure:

In other words, I can understand the meaning of an 'affirmation' ['Konstatierung'] only on and by way of a comparison with the facts, i.e., a carrying-out of the process required for the verification of all synthetic propositions. But whereas in all other synthetic statements [hypothesis: $\left[\begin{array}{c} \text{synthetic} \\ \text{und. } \neq_t^{\text{process}} \text{ ver.} \end{array} \right]$], establishing the meaning and establishing the truth are separate, clearly distinguishable processes [$\text{und. } \neq_t^{\text{process}} \text{ ver.}$], in observation statements [Konstatierungen: $\left[\begin{array}{c} \text{synthetic} \\ \text{und. } =_t^{\text{process}} \text{ ver.} \end{array} \right]$] they coincide [$\text{und. } =_t^{\text{process}} \text{ ver.}$], just as they do [$\text{und. } =_t^{\text{process}} \text{ ver.}$] in analytic judgements [$\left[\begin{array}{c} \text{analytic} \\ \text{und. } =_t^{\text{process}} \text{ ver.} \end{array} \right]$]. However different the 'affirmations' ['Konstatierungen'] may be from analytic propositions [\neq in D1], they have this in common, that in both the process of understanding is at the same time the process of verification [= in D2]. Along with their meaning I simultaneously grasp their truth [$\text{und. } =_t^{\text{process}} \text{ ver.}$]. To ask of an affirmation [Konstatierung] whether I might perhaps be mistaken about its truth, makes no more sense than with a tautology. Both have absolute validity [= in D1]. The analytical or tautological proposition, however, is at the same time devoid of content, whereas the observation statement gives us the satisfaction of a genuine acquaintance with reality [\neq in D2].

Konstatierungen are synthetic statements like *hypotheses*: identity / coincidence with respect to the first dimension D1. But they are completely different with respect to the second dimension D2. If we look at the relation

between *Konstatierungen* and *tautologies* we observe a similar situation but in reverse order: We have inequality in the first dimension but coincidence in the second one. In such an overall structure we call the central structure which links the left and the right structure up to identity with respect to two different dimensions *two-dimensionally structured intermediate cases*:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \neq \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

It is not clear whether such structures can be used to create a full-blooded *theory* of knowledge. But it is for sure that this was not Schlick's *philosophical* intention:

These moments of fulfilment and combustion are of the essence. From them comes all the light of knowledge. And it is this light for whose source the philosopher is actually asking, when he seeks the foundation of all knowledge. (ibid, p. 387)³²

4. KANT, FREGE, SCHLICK AND THEORETICAL VS. PHILOSOPHICAL PERSPECTIVES

Let us repeat *the general structure of two-dimensionally structured intermediate cases*: Let *A* and *B* be two expressions which differ in meaning completely but can be analyzed with respect two different dimensions D1 and D2. The grammatical or logical form of *A* and *B* is not fixed. It could be proper names, expressions denoting properties, expressions denoting *n*-ary relations, sentences and other expressions of natural languages. *A* and *B* can be represented by using numbers, individual constants, *n*-ary predicates, propositional constants or variables etc. Let A_1 (B_1) be the expression denoting the meaning of *A* (*B*) in the first dimension and A_2 (B_2) be the expression denoting the meaning of *A* (*B*) in the second dimension. There are plausible reasons that a reduction of these two dimensions to one is not intended. We take $A_i = B_i$ as an abbreviation that the expressions or the meaning of the expression *A* and *B* *coincide* in dimension *i*. We take $A_i \neq B_i$

³² „Diese Augenblicke der Erfüllung und des Verbrennens sind das Wesentliche. Von ihnen geht alles Licht der Erkenntnis aus. Und dies Licht ist es eigentlich, nach dessen Ursprung der Philosoph fragt, wenn er das Fundament alles Wissens sucht.“ (SCHLICK, 2008, p. 514)

as an abbreviation that the expressions or the meaning of the expressions A and B are *different* in dimension i . The general form to represent any expression A is $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$. In the case of B we take $\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$. It is very important that the structures of A_1 (B_1) and A_2 (B_2) need not be of the same grammatical or logical type. In the case of Schlick's *Konstatierungen* we took A_1 as a *property* of sentences: "... is synthetic" or a sentence containing an expression denoting a *property*: "'Here now yellow' is synthetic." But B_2 can be interpreted as a *binary relation*: "... is identical with ..." ("... coincide with ..") or a sentence containing an expression denoting a *binary relation*: "The verification of 'Here now yellow' consists with the process of understanding of 'Here now yellow'."

The following structure shows that the meanings of A and B is *completely different*, i.e., different in both dimensions:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \neq \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

This holds in particular if the two expression are dichotomic. In this situation we can link A and B by offering an intermediate case. Because we have to keep apart our two dimensions there are two candidates for that: $\begin{bmatrix} A_1 \\ B_2 \end{bmatrix}$ and $\begin{bmatrix} B_1 \\ A_2 \end{bmatrix}$:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ B_2 \end{bmatrix} \neq \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \neq \begin{bmatrix} B_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

In Schlick's case the second version would be something like the *anti-Konstatierung*:

$$\left[\begin{array}{c} \text{analytic} \\ \text{verification} \neq_{\text{PROCESS}} \text{understanding} \end{array} \right].$$

In Max (2020) I show that it is possible to reconstruct Kant's *synthetic judgments a priori* as specific intermediate cases within my two-dimensional language of analysis. Let me summarize the result. Kant motivation was to answer the question "How is metaphysics as science possible?" To show this possibility he needed a new type of judgments. If we identify the analytic-

synthetic-distinction with the distinction between a priori and a posteriori and if we put them into one and the same dimension then we get this situation:

$$(\text{analytic \& a priori}) \neq (\text{synthetic \& a posteriori}).$$

This represents the classical dichotomic distinction between truths of reason (analytic & a priori) and truths of fact (synthetic & a posteriori). But Kant's idea was to assume that the analytic-synthetic-distinction and the distinction between a priori and a posteriori do not belong to the same dimension:

D1: dichotomy 1 with respect to *the objects of judgments*: analytic – synthetic

D2: dichotomy 2 with respect to *the source of judgements*: a priori – a posteriori.

Now we represent truths of reason (analytic a priori judgments) and truths of facts (synthetic a posteriori judgments) and their complete distinctness in the following form:

$$\begin{bmatrix} \text{analytic} \\ \text{a priori} \end{bmatrix} \neq \begin{bmatrix} \text{synthetic} \\ \text{a posteriori} \end{bmatrix}.$$

But metaphysics should have (a) its own class of judgments which, additionally, (b) are able to link the two given distinct classes. This is the basis of bridging rationalism with empirism. If we look at this problem from our perspective of looking for intermediate cases it is easy to offer two candidates:

1. $\begin{bmatrix} \text{synthetic} \\ \text{a priori} \end{bmatrix}$: synthetic judgments a priori and
2. $\begin{bmatrix} \text{analytic} \\ \text{a posteriori} \end{bmatrix}$: analytic judgments a posteriori.

Clearly, Kant take the first alternative

$$\begin{bmatrix} \text{analytic} \\ \text{a priori} \end{bmatrix} \neq \begin{bmatrix} \text{synthetic} \\ \text{a priori} \end{bmatrix} = \begin{bmatrix} \text{synthetic} \\ \text{a posteriori} \end{bmatrix}$$

and argues explicitly against the second one:

$$\left[\begin{array}{l} \text{analytic} \\ \text{a priori} \end{array} \right] = \left[\begin{array}{l} \text{analytic} \\ \text{a posteriori} \end{array} \right] \neq \left[\begin{array}{l} \text{synthetic} \\ \text{a posteriori} \end{array} \right]^{33}$$

The question “Do analytic judgments a priori represent a new class of judgments which is simply distinct from or equal to one of the other classes” is deceptive. The seemingly possible answer “both” is not complete. The distance to the left and to the right neighbor is structurally the same. We have identity (coincidence) and inequality in both cases. But the main point is that we have to take into account and cannot neglect the difference between both dimensions. We have equality to the left in the *first* and equality to the right in the *second* dimension or the other way around! There is no chance to reduce the whole construction to a one-dimensional one. A piece of music in two voices where each sound is an interval sound consisting of different tones of both voices is not simply consisting of one melody!

How can we bring Frege into our play? It is well-known that Kant’s opinion is that equations like $5 + 7 = 12$ are synthetic judgments a priori. Frege’s program throughout his life was to show that equations are analytic and, therefore, a priori. So there seems to be no agreement with respect to the *content* of their positions. But with respect to our language of analysis we are able to show that Frege follows Kant *methodologically*. He has to answer the question “How is logic as science possible?” Are there any objects that are characteristic for logic? Frege (1918/19) starts with two disjoint classes of things: *things of the outer world* (*A*) and *ideas* = things of the inner world except decisions (*B*). Like in Kant Frege postulates two independent dimensions to characterize these things:

D1: the *perceivableness-dimension*: things can be perceived by senses (A_1) resp. things cannot be perceived by senses (B_1)

D2: the *bearer-dimension*: things are independent of a bearer (“needs no bearer to the contents of whose consciousness to belong”, Frege 1956, 302), they are *objective* (A_2) resp. things are dependent on a bearer, they are *subjective* (B_2).

³³ We will come back to this point in a moment because we know that *Konstatierungen* coincide with tautologies (analytic sentences) in one dimension.

We can represent (*A*) things of the outer world (objective things which we can perceive by sense) and (*B*) ideas (subjective things which we cannot perceive by sense) and their complete distinctness in an analogous form:

$$\left[\begin{array}{c} \text{perceivable by senses} \\ \text{objective} \end{array} \right] \neq \left[\begin{array}{c} \text{not perceivable by senses} \\ \text{subjective} \end{array} \right].$$

Again, we have two candidates for two-dimensionally structured intermediate cases (IC):

(IC1)

$$\left[\begin{array}{c} \text{perceivable by senses} \\ \text{objective} \end{array} \right] \neq \left[\begin{array}{c} \text{not perceivable by senses} \\ \text{objective} \end{array} \right] = \left[\begin{array}{c} \text{not perceivable by senses} \\ \text{subjective} \end{array} \right] \neq \left[\begin{array}{c} \text{perceivable by senses} \\ \text{subjective} \end{array} \right].$$

and

(IC2)

$$\left[\begin{array}{c} \text{perceivable by senses} \\ \text{objective} \end{array} \right] = \left[\begin{array}{c} \text{perceivable by senses} \\ \text{subjective} \end{array} \right] \neq \left[\begin{array}{c} \text{not perceivable by senses} \\ \text{subjective} \end{array} \right].$$

There is no hint that Frege was thinking about (IC 2). Because Frege is considering *sentences* here he call the *sense* [*Sinn*] of a sentence *the thought of this sentence*. Here is Frege's characterization of *thoughts* supplemented by our representations in square brackets:

So the result seems to be: *thoughts* $\left[\begin{array}{c} \text{not perceivable by senses} \\ \text{objective} \end{array} \right]$ are neither things of the outer world $\left[\begin{array}{c} \text{perceivable by senses} \\ \text{objective} \end{array} \right]$ nor ideas $\left[\begin{array}{c} \text{not perceivable by senses} \\ \text{subjective} \end{array} \right]$.

A third realm must be recognized. What belongs to this, corresponds with ideas, [= in D1] in that *it* cannot be perceived by the senses, but with things, [= D2] in that it needs no bearer to the contents of whose consciousness to belong. (ibid., p. 302)³⁴

³⁴ „So scheint das Ergebnis zu sein: Die Gedanken $\left[\begin{array}{c} \text{nicht mit den Sinnen wahrnehmbar} \\ \text{objektiv} \end{array} \right]$ sind weder Dinge der Außenwelt $\left[\begin{array}{c} \text{mit den Sinnen wahrnehmbar} \\ \text{objektiv} \end{array} \right]$, noch Vorstellungen $\left[\begin{array}{c} \text{nicht mit den Sinnen wahrnehmbar} \\ \text{subjektiv} \end{array} \right]$. / Ein drittes Reich muß anerkannt werden. Was zu diesem gehört, stimmt mit den Vorstellungen darin überein, [= in D1] daß es nicht mit den Sinnen wahrgenommen werden kann, mit den Dingen aber darin, [= in D2] daß es keines Trägers bedarf, zu dessen Bewußtseinsinhalte es gehört.“ (FREGE, 1918/19, p. 69).

But Frege is not only interested in showing the possibility of logic as science. This is a *philosophical* enterprise. But Frege has also a *theoretical* interest. He invented a logical *theory* with the thought as an undefined basic term or a principle which we call *bivalence* today. A consequence is that we could continue to represent the thought as a two-dimensional construction of the form $\begin{bmatrix} \text{the true} \\ \text{the false} \end{bmatrix}$. But the neighbors (the two possible meanings) could be represent as one-dimensional expressions:

$$\begin{bmatrix} \text{the false} \\ \text{the false} \end{bmatrix} \neq \begin{bmatrix} \text{the true} \\ \text{the false} \end{bmatrix} = \begin{bmatrix} \text{the true} \\ \text{the true} \end{bmatrix}$$

reduced to

$$\begin{bmatrix} \text{the false} \\ \text{the false} \end{bmatrix} - \begin{bmatrix} \text{the true} \\ \text{the false} \end{bmatrix} - \begin{bmatrix} \text{the true} \\ \text{the true} \end{bmatrix}.$$

With respect to this new type of structure there is no reason to argue against the opposite possibility $\begin{bmatrix} \text{the false} \\ \text{the true} \end{bmatrix}$, $\begin{bmatrix} \text{the true} \\ \text{the false} \end{bmatrix}$ and $\begin{bmatrix} \text{the false} \\ \text{the true} \end{bmatrix}$ are *opposite thoughts* of each other. We can transform the two forms into each other by using negation.³⁵

The important point in our context is that Frege uses his term “the thought” [„der Gedanke“] twofold: In the context of *things of the outer world* and *ideas* he discusses themes which belong in the context of *philosophy*. Here we find his arguments against psychologism. But the distinction between sense [Sinn] and meaning [Bedeutung] is relevant for the later versions of his *Begriffsschrift* (*Basic Laws of Arithmetic*). Because Frege was convinced that logic has to do with *content* and is not a purely formal game he established *the thought* as an *undefined basic term* as part of his logical *theory*. To know that a term is an undefined basic one

³⁵ The reader may be more familiar with the representation using truth-tables: $\begin{matrix} p & p \\ \text{true} & \text{false} \end{matrix}$ and $\begin{matrix} p & p \\ \text{false} & \text{true} \end{matrix}$

with $\neg \text{true} = \text{false}$ and $\neg \text{false} = \text{true}$. This holds initially only for elementary propositions. In MAX, 2020 I have shown how this approach can be extended to a full 2-dimensional logic which is equivalent to classical propositional logic. In MAX, 2003 I showed that by changing the inner structure of the thought we will get non-classical logics.

presupposes that the theory already exists. Kant did not make this kind of distinction between philosophy on the one hand and science (theory) on the other. Showing the possibility of metaphysics as science is already an theoretical enterprise. But if theories have their own rights then showing the possibility of a theory is not the theory itself.

The discussion between Schlick and Einstein about space-time coincidences is a discussion about *understanding* or *interpreting* an already existing theory. It is not a discussion about an undefined term of the theory of relativity. Einstein was able to give up the idea of such coincidences without being forced to withdraw his theory of relativity. If we understand epistemology as a true *theory* of science then Schlick was trivially not successful because he never developed an epistemology with *coincidence* as a basic term. But if epistemology is interpreted as *philosophy* of science we get another picture. We can discuss whether *coincidence* could be an interesting proposal of understanding and interpreting theories in particular and discussing philosophical questions in general. I have tried to show that thinking about *coincidences* and *Konstatierungen* leads us to a multi-dimensional approach which can be fruitful in methodological respects.

Looking at Schlick's *Konstatierungen* we immediately observe significant disagreements with Kant's *synthetic a priori judgments* if we draw attention to the sub-expressions in the two-dimensionally structured intermediate cases:

$$\left[\begin{array}{c} \text{synthetic} \\ \text{a priori} \end{array} \right] \text{ vs. } \left[\begin{array}{c} \text{synthetic} \\ \text{understanding} = \underset{t}{\text{process}} \text{ verification} \end{array} \right].$$

Compare, e.g., “5 + 7 = 12” and “All bodies are extended.” with “Here now yellow.”!? It is clear that “synthetic” in the first construction does not mean the same like “synthetic” in the second one. As an empirist Schlick criticizes the use of “a apriori” by Kant and the enormous variety of interpretations of this term by the Neokantians again and again. He introduces a new term associated mainly with science—*verification*—and another one mainly associated with philosophy—*understanding*—together with considering processes, actions of pointing etc. In spite of all these differences Schlick follows—like Frege—Kant *methodologically*. But there is a very important new aspect in the inner structure

of his *Konstatierungen*: We can interpret “... is synthetic” and “... is a priori” as properties which could be represented using *unary* functions. But for representing the process-related identity (process-coincidence with respect to one moment) we have to use a *binary* relation. In spite of their different contents these two-dimensionally structured intermediate cases differ in their *inner structure*!

Transcending Kant’s construction of his *synthetic a priori judgments* Schlick opens the perspective to look for more complex inner structures. This concerns only the complexity of *A* and *B* within two-dimensionally structured intermediate cases of the form $\begin{bmatrix} A \\ B \end{bmatrix}$. E.g., not only *B* but also *A* could be an expression denoting a *relation*. I was not able to find some hint that Schlick look for intermediate cases which are *n-dimensionally structured* with $n > 2$. And we could consider nested structures like $\left[\left[\quad \right] \right]$.

Many interpreters—starting with Carnap—have trouble with Schlick’s *Konstatierung*, because they suppose that Schlick intends to find an undefined basic term for his epistemology as a *theory* of knowledge. Uebel (2020) speaks throughout his work of Schlick’s “theory of affirmation [Konstatierung]”. I think that this was not Schlick’s primary and also not his secondary intention. “All great attempts at establishing a theory of knowing arise from the problem of the reliability of human cognition, and this question again arises from the wish for absolute certainty of knowledge.”³⁶ (SCHLICK, 1979, p. 370). To characterize his topic—“the foundation of knowledge”—Schlick goes backwards stepwise starting with “a theory of knowing”, continuing with “establish” [“begründen”] it, with “the problem” [“der Frage”] and ending with “the wish” (“den Wunsche”) “for absolute certainty of knowledge” (“nach absoluter Gewißheit der Erkenntnis”). This is clearly a *philosophical* enterprise which cannot be realized by providing “a theory of knowledge”. Maybe our structured

³⁶ „Alle großen Versuche der Begründung einer Theorie des Erkennens entspringen aus der Frage nach der Sicherheit menschlichen Wissens, und diese Frage wiederum entspringt aus dem Wunsche nach absoluter Gewißheit der Erkenntnis.“ (SCHLICK, 2008, p. 487)

reconstruction of Schlick's *Konstatierungen* looks more *theoretical* than many other investigations which are intended to be *philosophical* but show many uses of "theory" and "theoretical". By contrast, using *structured intermediate cases* in our language of analyses to *interpret* given texts should not be confused with a genuine *theory* of interpretation.

A main source of our failure to understand is that we do not *command a clear view* of the use of our words.—Our grammar is lacking in this sort of perspicuity. A perspicuous representation produces just that understanding which consists in 'seeing connexions'. Hence the importance of finding and inventing *intermediate cases*.

The concept of a perspicuous representation is of fundamental significance for us. It earmarks the form of account we give, the way we look at things. (Is this a ‚Weltanschauung‘?) (WITTGENSTEIN, 1986, p. 49: PI 122)³⁷

5. REFERENCES

MORITZ SCHLICK

SCHLICK, M. "Die Grenzen der naturwissenschaftlichen Begriffsbildung", *Vierteljahresschrift für wissenschaftliche Philosophie und Soziologie*. gegründet von Richard Avenarius, in Verbindung mit Friedrich Jodl und Alois Riehl, herausgegeben von Paul Barth. Leipzig O. R. Reisland, 1910, p. 121-142.

SCHLICK, M. "Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie", *Die Naturwissenschaften* 5, 1917a p. 161-167; p. 177-186.

SCHLICK, M. *Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie*, Berlin: Springer, 1917b, 1. Auflage. IV+63.

SCHLICK, M. *Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitäts- und Gravitationstheorie*, zweite, stark vermehrte Auflage, Berlin: Springer, 1919, VI+86.

³⁷ „Es ist eine Hauptquelle unseres Unverständnisses, daß wir den Gebrauch unserer Wörter nicht *übersehen*. – Unserer Grammatik fehlt es an Übersichtlichkeit. – Die übersichtliche Darstellung vermittelt das Verständnis, welches eben darin besteht, daß wir die ›Zusammenhänge sehen‹. Daher die Wichtigkeit des Findens und des Erfindens von *Zwischengliedern*. Der Begriff der übersichtlichen Darstellung ist für uns von grundlegender Bedeutung. Er bezeichnet unsere Darstellungsform, die Art, wie wir die Dinge sehen. (Ist dies eine ›Weltanschauung‹?)“ (WITTGENSTEIN, 1984, p. 302: PU 122).

- SCHLICK, M. *Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie*, Berlin: Springer 1917, dritte, vermehrte und verbesserte Auflage, Berlin: Springer, 1920. VI+90.
- SCHLICK, M. "Kritizistische oder empiristische Deutung der neuen Physik?" *Kant-Studien* 26, Heft 1-2, 8, 1921, p. 96-111.
- SCHLICK, M. *Space and Time in Contemporary Physics. An Introduction to the Theory of Relativity and Gravitation*, rendered into English by Henry L. Brose, with an introduction by F. A. Lindemann, Oxford: Clarendon Press/New York: Oxford University Press 1920. X+89. English translation of Schlick, 1920. (1920engl).
- SCHLICK, M. *Raum und Zeit in der gegenwärtigen Physik. Zur Einführung in das Verständnis der allgemeinen Relativitätstheorie*, Berlin: Springer 1917, vierte, vermehrte und verbesserte Auflage, Berlin: Springer 1922. VI+107. Reprinted in SCHLICK, M. *Über die Reflexion des Lichtes in einer inhomogenen Schicht. Raum und Zeit in der gegenwärtigen Physik*, edited and commented by Fynn Ole Engler & Matthias Neuber, Springer: Wien, New York, (1922/2006), p. 121-345.
- SCHLICK, M. Über das Fundament der Erkenntnis. *Erkenntnis* 4 (1934) Heft 2 vom 10. Juli, 79-99. Reprinted in Moritz Schlick. *Die Wiener Zeit. Aufsätze, Beiträge, Rezensionen 1926-1936*; edited and commented by Johannes Friedl & Heiner Rutte. Springer: Wien, New York, 1934/2008, p. 479-514.
- SCHLICK, M. "Introduction [to « Sur le fondement de la connaissance »]". In: *Sur le fondement de la connaissance*. Traduction / Du Général Vouillemin / Ancien élève de l'École Polytechnique, Paris / Hermann & Cie, Éditeurs / 6, Rue de la Sorbonne, 6 / 1935, 3-7. Reprinted in SCHLICK, M. *Die Wiener Zeit. Aufsätze, Beiträge, Rezensionen 1926-1936*; edited and commented by Johannes Friedl & Heiner Rutte. Springer: Wien, New York, 2008, p. 645-653. (1935fr-A/2008-A).
- SCHLICK, M. Sur le fondement de la connaissance. In: *Sur le fondement de la connaissance*. Traduction / Du Général Vouillemin / Ancien élève de l'École Polytechnique, Paris / Hermann & Cie, Éditeurs / 6, Rue de la Sorbonne, 6 / 1935, p. 8-34. French translation of SCHLICK, 1934. (1935fr-B/1934).
- SCHLICK, M. "Les énoncés et les faits". In: *Sur le fondement de la connaissance*. Traduction / Du Général Vouillemin / Ancien élève de l'École Polytechnique, Paris / Hermann & Cie, Éditeurs / 6, Rue de la Sorbonne, 6 / 1935, p. 35-43. (1935fr-C/2008-C).
- SCHLICK, M. "Sur les « constatations »". In: *Sur le fondement de la connaissance*. Traduction / Du Général Vouillemin / Ancien élève de l'École Polytechnique, Paris / Hermann & Cie, Éditeurs / 6, Rue de la Sorbonne, 6 / 1935, p. 44-54. Reprinted in SCHLICK, M. *Die Wiener Zeit. Aufsätze, Beiträge, Rezensionen 1926-1936*; edited and commented by Johannes Friedl & Heiner Rutte. Springer: Wien, New York, 2008, p. 657-671. (1935fr-D/2008-D).

- SCHLICK, M. "On the Foundation of Knowledge". In: *Logical Positivism*, ed. by Alfred Jules Ayer, West Port: Greenwood Press, 1959; translated from SCHLICK, 1934 by David Rynin. (1959engl/1934).
- SCHLICK, M. On the Foundation of Knowledge. In: Schlick, *Philosophical Papers* vol. II (1925-1936), ed. by Peter Heath, Wilfrid Sellars, Herbert Feigl & Mary Brodbeck, 370-387; English translation of Schlick (1934) by Peter Heath. (1979engl/1934).
- SCHLICK, M. "Introduction". In: SCHLICK, M. *Philosophical Papers* vol. II (1925-1936), ed. by Peter Heath, Wilfrid Sellars, Herbert Feigl & Mary Brodbeck, p. 405-407; translated from unpublished German original by Peter Heath. (1979engl-a).
- SCHLICK, M. "On 'Affirmations'" In: SCHLICK, M. *Philosophical Papers* vol. II (1925-1936), ed. by Peter Heath, Wilfrid Sellars, Herbert Feigl & Mary Brodbeck, p. 407-413; translated from unpublished German original by Peter Heath. (1979engl-b).

FURTHER REFERENCES

- CARNAP, R. *The Philosophy of Rudolf Carnap*. (The Library of Living Philosophers XI), ed. P. A. Schilpp, Illinois: Open Court, 1997³.
- CHAPUIS-SCHMITZ, D. "What is Physically Real? Schlick vs. Einstein", *Beiträge der Österreichischen Ludwig Wittgenstein Gesellschaft*, Kirchberg am Wechsel: ÖLWG, 2005, p. 41-43.
- DAVIDON, D. "Empirical content". *Grazer Philosophische Studien* 16/17: p. 471-489. Repr. In: LEPORE, E. (ed.) *Essays on truth and interpretation*, Oxford: Oxford University Press, 1987.
- ENGLER, F. O. "Moritz Schlick und Hans Reichenbach über die Eindeutigkeit der Zuordnung, die Gründe diese aufzugeben und die heuristische Stärke eines Empirismus mit begriffskonstitutiven Prinzipien". In: OLE ENGLER, F.; IVEN, M. (eds.), *Moritz Schlick. Leben, Werk und Wirkung*, Berlin, 2008, p. 131-191.
- ENGLER, F. O. "Über das erkenntnistheoretische Raumproblem bei Moritz Schlick, Wilhelm Wundt und Albert Einstein", In STADLER, F.; WENDEL, H. J. (Eds.), *Stationen: dem Philosophen und Physiker Moritz Schlick zum 125. Geburtstag*, Wien [u.a.]: Springer, 2009, p. 107-145.
- ENGLER, F. O. "Koinzidenzen, Gestalten und Denkstile: Moritz Schlick und Ludwik Fleck über die Objektivität am Fundament der Erkenntnis und die Bedeutung der Wissenschaftsgeschichte" In: ENGLER, F. O.; IVEN, M. (Eds.), *Moritz Schlick: Ursprünge und Entwicklungen seines Denkens*. Berlin: Parerga, 2010, p. 339-386.
- ENGLER, F. O.; RENN, J. *Gespaltene Vernunft: vom Ende eines Dialogs zwischen Wissenschaft und Philosophie*. Berlin: Matthes & Seitz, 2018.

- FABIAN, R. *Inventarverzeichnis des wissenschaftlichen Nachlasses von Moritz Schlick*, Wiener Kreis Archiv (Noord-Hollands Archief, Haarlem/NL), Haarlem/Graz, 2007.
- FERRARI, M. Der Philosoph als Schatzgräber. Moritz Schlick und das „Weltbild“ der Relativitätstheorie Einsteins“, *Beiträge der Österreichischen Ludwig Wittgenstein Gesellschaft, Kirchberg am Wechsel: ÖLWG*, 2005, p. 66-69.
- FREGE, G. Der Gedanke. Eine logische Untersuchung. Beiträge zur Philosophie des deutschen Idealismus 2, 1918-1919a, p. 58-77. <http://www.gavagai.de/HHP32.htm>.
- FREGE, G. *The Thought: A Logical Inquiry*. Translation of FREGE 1918/19. Mind, New Series, vol. 65, No. 259, 1956, 289-311. <http://inescrespo.altervista.org/nyu/Frege1918.pdf>. (1956engl). <http://www.jstor.org/stable/2251513>.
- FRIEDL, J. *Konsequenter Empirismus. Die Entwicklung von Moritz Schlicks Erkenntnistheorie im Wiener Kreis (Schlick-Studien vol. 3)*, Springer: Wien, New York, 2013, especially chapter 5.
- FRIEDL, J. "Erkenntnisbasis und Intersubjektivität. Moritz Schlick und Béla Juhos über Konstatierungen", *Kriterion – Journal of Philosophy*, 30, 3, 2016, p. 93-120.
- GLASSNER, E. "Was heißt Koinzidenz bei Schlick?" In: STADLER, F.; WENDEL, H. J.; GLASSNER, E. (eds.) *Stationen. Dem Philosophen und Physiker Moritz Schlick zum 125. Geburtstag. Schlick Studien*, vol. 1. Springer, Vienna, 2009, p. 146-166. https://doi.org/10.1007/978-3-211-71581-9_5.
- HEMPEL, C. G. "On the logical positivists theory of truth". *Analysis* 2, 1935, p. 49-59.
- KANT, I. *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können*. Riga: Johann Friedrich Hartknoch. 1783. Fünfte Auflage mit Einleitung, Beilagen, Personen- und Sachregister herausgegeben von Karl Vorländer. Leipzig: Verlag von Felix Meiner, 1913. <https://digital.ub.uni-leipzig.de/mirador/index.php#b2599508-dfdd-42c9-8896-12ed9c5dd645>.
- KANT, I. *Kritik der reinen Vernunft*. Nach der ersten und zweiten Originalausgabe herausgegeben von Jens Timmermann. Mit einer Bibliographie von Heiner Klemme. Hamburg: Felix Meiner Verlag, 1998. http://www.ciando.com/img/books/extract/3787321128_lp.pdf.
- KANT, I. *Critique of Pure Reason*. Translated and edited by Paul Guyer (University of Pennsylvania) & Allen W. Wood (Yale University). *The Cambridge edition of the works of Immanuel Kant*. Cambridge: University Press, 1998. <https://cpb-us-w2.wpmucdn.com/u.osu.edu/dist/5/25851/files/2017/09/kant-first-critique-cambridge-1m89prv.pdf>.
- KANT, I. *Prolegomena to Any Future Metaphysics. That Will Be Able to Come Forward as Science*. Translated and edited by Gary HATFIELD, University of

- Pennsylvania, Revised Edition, Cambridge: University Press, 2004.
<http://strangebeautiful.com/other-texts/kant-prolegomena-cambridge.pdf>
- MAHLER, C. "The Linguistic Pitfall for Konstatierungen (or Schlick's Foundation of Belief) as Language of Thought or Natural Language Sentences.", *Logos: Revista de Lingüística, Filosofía y Literatura* 27 (2), 2017, p. 246-252. <https://doi.org/10.15443/RL2719>.
- MAX, I. "Metasuche - Das (Er-)Finden von Zwischengliedern. Analysen zu ausgewählten Texten von Kant, Schlick, Aristoteles und Austin" In: *Philosophieren über Philosophie* (= Leipziger Schriften zur Philosophie, vol. 10), ed. by Richard Raatzsch, Leipzig: Leipziger Universitätsverlag, 1999, p. 241-262.
- MAX, I. "Freges Gedanke und seine nichtklassischen Verwandten". In: MAX, I. (ed.) *Traditionelle und moderne Logik. Zum 65. Geburtstag von Lothar Kreiser*, Leipzig: Universitätsverlag, 2003, p. 149-179.
- MAX, I. "Similar Ways of Creating Thirdness: Kant's 'Synthetic Judgments a priori' and Frege's 'Thoughts' as Intermediate Cases. *Revista de Filosofia Moderna e Contemporânea*, Brasília, v.8, n.2, ago 2020, p. 79-118. <https://doi.org/10.26512/rfmc.v8i2.35848>.
- MAX, I. "Programmatisches zur Komplexität des Verhältnisses zwischen Schlick und Wittgenstein" To be published in: LEMKE, M.; LESCHKE, K.; PETERS, F.; WUNSCH, M. (orgs.) *Der Wiener Kreis und sein philosophisches Spektrum. Beiträge zur Kulturphilosophie, Metaphysik, Philosophiegeschichte, Praktischen Philosophie und Ästhetik* (= Studien zur Philosophie des 20. und 21. Jahrhunderts, vol. 1), Heidelberg: Metzler. (forthcoming)
- MAX, I.; BORCHERS, R.; FRANKE-REDDIG, J. LEON BAUER, P. "Konsequentes Philosophieren im Lichte empirischer Wissenschaften und Logik. Drei neue Bände der Nachgelassenen Schriften der Moritz Schlick Gesamtausgabe. Literaturbericht zu den Bänden II/1.1", *Frühe erkenntnistheoretische Schriften*, II/1.3 *Vorlesungen und Aufzeichnungen zur Logik und Philosophie der Mathematik* und II/2.1 *Naturphilosophische Schriften. Manuskripte 1910-1936*. To be published in *Zeitschrift für philosophische Forschung*. (forthcoming).
- NEUBER, M. *Die Grenzen des Revisionismus. Schlick, Cassirer und das ‚Raumproblem‘*, Wien & New York: Springer, 2012.
- NEUBER, M. "Schlick und die ‚Wende der Philosophie‘ - Vom kritischen Realismus zum logischen Empirismus (und wieder zurück?)" In: NEUBER, M. (ed.) *Husserl, Cassirer, Schlick. „Wissenschaftliche Philosophie im Spannungsfeld von Phänomenologie, Neukantianismus und logischem Empirismus* (= Veröffentlichungen des Instituts Wiener Kreis, Switzerland: Springer, 2016, p. 207-235
- OBERDAN, T. "Postscript to protocols: Reflections on empiricism" In: GIÈRE, R.; RICHARDSON, A. (eds.), *Origins of logical empiricism*, Minneapolis: University of Minnesota Press, 1996, p. 269-91.
- OBERDAN, T. "The Vienna Circle's 'anti-foundationalism'". *British Journal for the Philosophy of Science* 49, 1998, p. 297-308.

- OBERDAN, T. "Deconstructing protocols: Reply to Uebel". *British Journal for the Philosophy of Science* 50, 1999, p. 301-314.
- RYCKMAN, T. "Conditio sine qua non? >Zuordnung< in the early epistemologies of Cassirer and Schlick". *Synthese* 88, 1991, p. 57-95.
- RYCKMAN, T. "P(oint)-C(oincidence)-Thinking: The Ironical Attachment of Logical Empiricism to General Relativity (and Some Lingering Consequences)". *Studies in History and Philosophy of Science* 23 (3), 1992, p. 471-497.
- RYCKMAN, T. *The Reign of Relativity. Philosophy in Physics 1915-1925*, New York: Oxford University Press, 2005.
- SELLARS W. *Empiricism and the Philosophy of Mind*. Minnesota Studies in the Philosophy of Science Volume I, University of Minnesota Press, 1956.
- UEBEL, T. "Anti-foundationalism and the Vienna Circle's revolution in philosophy". *British Journal for the Philosophy of Science* 47, 1996, p. 415-440.
- UEBEL, T. "Protocols, affirmations, and foundations: Reply to Oberdan". *British Journal for the Philosophy of Science* 50, 1999, p. 297-300.
- UEBEL, T. *Empiricism at the Crossroads. The Vienna Circle's Protocol-Sentence Debate*. Chicago: Open Court, 2007.
- UEBEL, T. "Schlick und die empirische Basis der Wissenschaft". In: NEUBER, M. (ed.) *Husserl, Cassirer, Schlick. „Wissenschaftliche Philosophie im Spannungsfeld von Phänomenologie, Neukantianismus und logischem Empirismus (= Veröffentlichungen des Instituts Wiener Kreis)*, Switzerland: Springer, 2016, p. 237-261.
- UEBEL, T. "Schlick and Wittgenstein: The Theory of Affirmations Revisited". *Journal of the History of Philosophy*, vol. 58, Number 1, January 2020, p. 141-166.
- WITTGENSTEIN, L. *Tractatus logico-philosophicus, Tagebücher 1914–1916, Philosophische Untersuchungen* [PU]. Werkausgabe Band 1. Frankfurt am Main: Suhrkamp, 1984.
- WITTGENSTEIN, L. *Philosophical Investigations* [PI]. Edited and translated by Gertrude Elizabeth Margaret Anscombe, 3rd ed. Oxford: Basil Blackwell 1986.

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