REDECA
Revista Eletrônica do Departamento de Ciências Contábeis & Departamento de Atuária e Métodos Quantitativos da FEA

Order book, order flow and returns: evidence from the Brazilian stock exchange

Michel Alexandre[1]

Fernando Fagundes Ferreira[2]

Camilo Rodrigues Neto[3]

Resumo

Este trabalho possui como objetivo estudar os fatos estilizados do livro de ofertas dos papéis negociados na Bolsa de Valores de São Paulo (BOVESPA), assim como dos retornos engendrados pela dinâmica do livro de ofertas. Trabalhou-se com dados de junho/2006 a janeiro/2009 de uma amostra formada pelos vinte papéis mais negociados da BOVESPA. Os resultados empíricos corroboraram alguns fatos estilizados observados no estudo de papéis de outros países, mas refutaram outros.

Palavras-chave: Livro de ofertas; Econofísica; Bolsa de valores.

Abstract

The purpose of this study is to analyze the stylized facts of the order book of stocks traded in the São Paulo Stock Exchange (BOVESPA), as well as of the returns engendered by the order book dynamics. We used data from June 2006 to January 2009 regarding a sample composed by the twenty most traded stocks in BOVESPA. The empirical results corroborated some stylized facts observed in stocks of other countries, but refuted others.

Keywords: Order book; Econophysics; Stock market.

[1] Mestre em Modelagem de Sistemas Complexos pela Universidade de São Paulo. Doutorando em Ciências de Computação e Matemática Computacional pela Universidade de São Paulo.E-mail: michel.alsilva@gmail.com
[2] Professor Doutor da Universidade de São Paulo. E-mail: ferfff@usp.br
[3] Professor Doutor da Universidade de São Paulo. E-mail: camiloneto@usp.br

1 Introduction

Proposals (*orders*) to buy or sell a given number of shares can be of two basic types. If the order is immediately executed, it is a *market order*. Otherwise, it is stored in an *order book* for future execution, being called *limit order*. An order book is, thus, a list of limit orders available at a given instant of time, displaying some information about them – such as the maximum (minimum) price at which the investor is willing to buy (sell) the shares, the quantity of shares the investor is willing to negotiate and the time the order was registered.

Order book data became available in the second half of 1980s with the computerization of financial markets (Chakraborti et al, 2011), launching to researchers a challenge closely related to the concept of complex systems: to reproduce the aggregate behavior of financial markets – reflected in prices and returns – from the interaction among its most microscopic components. Indeed, the main motivation of the study of order books is to figure out the microfoundations of price variation. Price dynamics can be though as the result of the interplay between the structure of the order book and the flow of new orders (Bouchaud et al, 2002).

Since the seminal work of Biais et al (1995), many studies (e.g., Bouchaud et al, 2002; Potters and Bouchaud, 2003; Farmer et al, 2004; Bouchaud et al, 2009; Chakraborti et al, 2011) analyzed the statistical properties of order books. The main stylized facts found are the power law decay of incoming limit order prices, the peculiar shape of the order book, peaked away from the best quote, the non-trivial power law correlation of the transaction price signs, and the non linear impact of transacted volume on prices (Zaccaria et al, 2010). Nonetheless, other statistics seems to be of hard characterization. For example, while Bouchaud et al (2002) reports that the volume of orders follows a uniform distribution, other studies, revised in Chakraborti et al (2011), attest that it obeys a power law distribution.

Most of studies on order books refer to developed economies. There are few exceptions as, for instance, Dremin and Leonidov (2005) on the Russian stock market and Gu et al (2008) on Chinese stocks. However, analysis of order books in emerging markets can shed light on interesting points. There is evidence that the asset price dynamic presents some peculiarities in emerging markets. Duration of asset price cycles is similar in both developed and emerging markets, but the median peak-to-trough amplitudes for stock prices are larger for emerging markets (Canuto and Cavallari, 2013). An important question is whether differences between developed and emerging markets are observed also regarding the order book structure. In positive case, it means that the above-mentioned particular macro behavior of financial emerging markets is at least partially due to specificities at the micro level.

The purpose of this paper is to fill partially this gap analyzing the stylized facts of the São Paulo Stock Exchange (BOVESPA) order book, the main Brazilian stock market. Beside this introduction, it has more five parts. Section 2 brings some definitions and discusses how order book, order flow and returns are correlated. The data base is presented in Section 3. The next section shows the statistical regularities of the order book and the return stemmed from its dynamics. A market impact function, expressing the relationship between the transacted volume and return, is estimated in Section 5. Final considerations take the last section.

2 The order book: definitions

Consider all the offered orders in the order book at a given instant t – that is, orders registered at or before t and not yet executed/cancelled at this instant. Sorting in ascending order the prices of all sell orders offered at t, we have $a_{0,t}$, $a_{1,t}$, …, $a_{n,t}$. The smallest value of this series, $a_{0,t}$, in simply called *ask*. In a similar fashion, arranging in descending order the prices of all buy orders, we have $b_{0,t}$, $b_{1,t}$, …, $b_{m,t}$. The *bid* refers to the biggest buy price, $b_{0,t}$. The *spread* is equal to $a_{0,t} - b_{0,t}$ and is always positive. The midpoint m_t is equal to $(a_{0,t} + b_{0,t})/2$ and is widely used in the literature as a proxy for the share price. The *i-th gap* is equal to $a_{i,t} - a_{i-1,t}$ on the ask side and to $b_{i-1,t} - b_{i,t}$ on the bid side, being i equal or greater than 1.

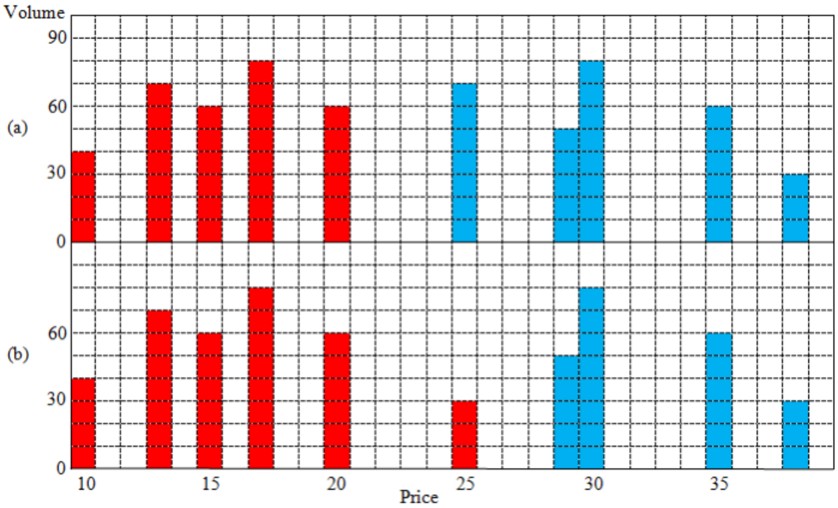
The queue of the order book at t Q_t corresponds to the set $\{b_{m,t},..., b_{0,t}, a_{0,t},..., a_{n,t}\} = \{q_{1,t}, \ ..., \ q_{m+n+2,t}\}$. Summing up the volume of all orders whose price is equal to $q_{i,t}$ we have $v_{i,t}$. The set V_t is equal to $\{v_{1,t}, \ ..., \ v_{m+n+2,t}\}$. The figure which plots Q_t and V_t is the *instantaneous format of the order book* at t. Suppose an order with price p and volume v is registered at time t. The *delta* of this order is equal to $p - a_{0,t}$ if it is a sell order or $b_{0,t} - p$ if it is a buy order. There are three possibilities:

1. $b_{0,t} \geq p$ (or $a_{0,t} \leq p$, in case of a sell order): the order is stored in the order book. The volume $v_{i,t}$ for $q_i = p$ is increased in v. There is neither transaction nor change in the price.
2. $b_{0,t} < p < a_{0,t}$: the order is stored in the order book. There is no transaction, but m_t changes (increases in case of a buy order or decreases in case of a sell order).
3. $a_{0,t} \leq p$ (or $b_{0,t} \geq p$, in case of a sell order): a transaction occurs at the best price and the order is nor registered in the order book. m_t will change just if v is equal or greater than the volume at the best price.

Figure 3.1 brings an example of the relationship between order book structure, order flow and price shift. At instant (a), the order book of a given stock has the format depicted in the superior part of the figure. A buy order of p = 25 and v = 100 is registered at this moment. The order book acquires the format expressed at instant (b) and the midpoint changes from 22.5 to 27. It is thus clear how price shifts stem from the interplay between the order book format and the order flow, whose statistical regularities will be analyzed in the section 4.

REDECA – Revista Eletrônica do Departamento de Ciências Contábeis &Departamento de Atuária e Métodos Quantitativos da FEA-PUC/SP

Figure 3.1: Order book format in two instants. Red (blue) columns refer to buy (sell) orders. The format of the order book changes from (a) to (b) after the register of an order of price 25 and volume 100.



Source: made by the authors.

3 The database

Our database comprises the BOVESPA twenty most traded stocks between June 1st, 2006 and January 31st, 2009. Together, they account for almost 35% of the total number of transactions (Table 3.1). For each order, we have the following information: type (buy or sell), price, volume, arrival time and negotiation time. We have also, for each transaction, the time the transaction was done, the traded volume, the traded price and an identification of the orders involved.

It is necessary to make an important remark about the database. We also have information about the orders registered in the order book which were not executed, that is, that were cancelled after some time. Nonetheless, we do not know the exact instant in which these orders were cancelled; it is known just the days they were registered in the order book. Bouchaud et al (2002) faced a similar problem. The choice made by the authors was to keep them until the moment they were sure these orders were still in the order book. However, we have two additional problems. First, most of these orders appear just one day in the order book. Thus, by this criterion, all these orders should be excluded from the study, as we do not know the precise moment of the day they were withdrawn from the order book. Second, an important information, the volume, is not provided in case of non executed orders. Hence, we decided to use just executed orders in this study.

4 Stylized facts

4.1 Order book

From our database, we reconstructed the order book of the analyzed stocks for all trading days between June 1st, 2006 and January 31st, 2009, during the period of continuous negotiation of BOVESPA (from 11:00 to 18:00 in the daylight saving time period and from 10:00 to 17:00 otherwise). Q_t and V_t were calculated considering a time interval Δt = 1 minute. Then we proceeded as follows:

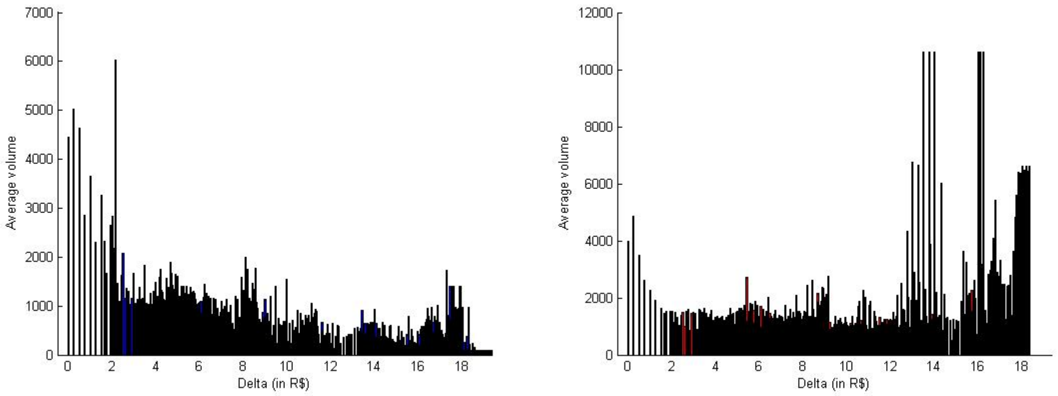
- Considering just the ask side of the order book, we computed $d_{i,t}^A = q_{i,t} - a_{0,t}$.
- All existing values of $d_{i,t}^A$ were queued in ascending order in the set $D^A = \{d_1^A, \quad ..., \quad d_n^A\}$. Observe that d_1^A is equal to zero.
- It was created the set $V^A = \{v_1^A, \quad ..., \quad v_n^A\}$, where $v_j^A = E[v_{i,t} | d_{i,t} = d_i^A]$ for j = 1,…, n.
- Plotting D^A and V^A, we have the *average shape of the order book,* or simply the average order book, on the ask side. The procedure is applied to the buy orders to compute D^B, V^B and the average order book on the bid side.

4.1.1 Average order book

Empirical studies (e.g., Biais et al, 1995; Chakraborti et al, 2011) show that the maximum of the average order book is observed away from the best price (delta equal to zero). The reason is that, although the order flow is maximum around the best price, orders whose deltas are very close to zero have greater chances of being executed and, thus, disappearing from the book (Bouchaud et al, 2002). Table 4.1 shows the volume offered on the best quote as a fraction of the maximum volume observed in the average order book, as well as the delta at which such maximum is located. It can be seen that there is a great dissimilarity of these data across stocks e between the different sides of the order book. In eight (seven) stocks, the maximum volume is offered at the best price in the ask (bid) side, refuting the results obtained for other stocks. The symmetry between ask and bid sides reported by Bouchaud et al (2002) is another feature which was not observed in the analyzed stocks. Figure 4.1 shows the average order book for PETR4.

REDECA – Revista Eletrônica do Departamento de Ciências Contábeis &Departamento de Atuária e Métodos Quantitativos da FEA-PUC/SP

Figure 4.1: Average order book for PETR4 on the ask (left) and bid (right) side.



Source: made by the authors

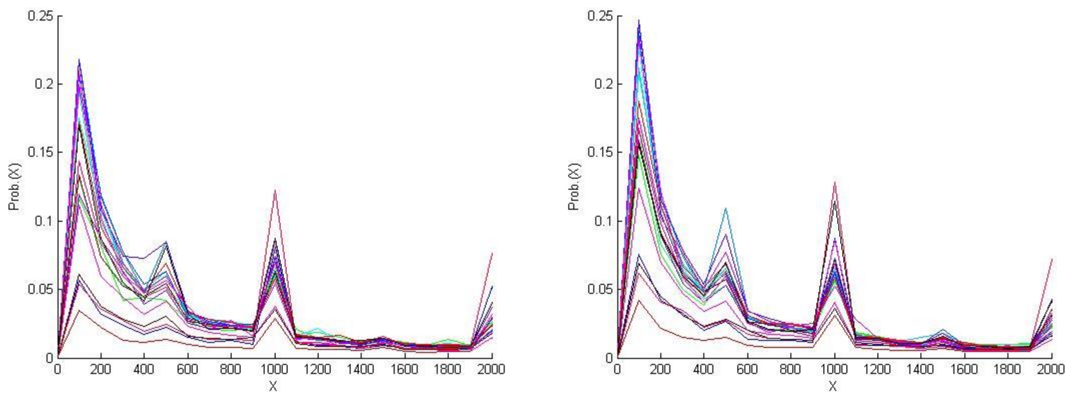
4.1.2 Volume at best price

The volume at the best price is an element of the order book which is closely related to the price dynamics. This is a measure of stock liquidity, and the higher this value, the higher should be the transacted volume in order to cause a price variation. Analyzing three stocks of the Paris stock market, Bouchaud et al (2002) found that the volume at the best quote follows a gamma distribution with $\gamma \approx 0.7 - 0.8$. The stocks of our sample presented a poor fitting to the gamma distribution. Performing a chi-square test, the hypothesis of gamma distribution was rejected at the 5% significance level for all of them. The distribution of the volume at the best quote for the analyzed stocks is presented in Figure 4.2. However, their cumulative distribution is very well represented by the following equation:

$$P(x > X) \approx e^{-a(\log(X))^{b}} \quad (4.1),$$

with b ranging from 2.6 and 5.4 (Table 4.2). Nonetheless, for values bellow 3.6, the fit was unsatisfactory. Figure 4.3 shows the fit of empirical data of PETR4 to equation 4.1.

Figure 4.2: Distribution of the volume at the ask (left) and the bid (right).



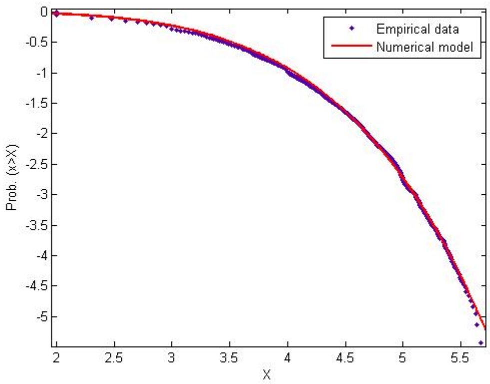
Source: made by the authors

Table 4.2: Fit of empirical data to equation 4.1

Stock	Volume at ask		Volume at bid	
	b	Adjusted R^2	b	Adjusted R^2
PETR4	4.9	0.9981	4.5	0.9981
VALE5	4.5	0.9963	3.6	0.9534
BBDC4	4.2	0.9974	3.8	0.9937
ITAU4	3.6	0.9876	4.3	0.9918
GGBR4	3.9	0.9950	4.2	0.9960
CSNA3	4.6	0.9883	4.2	0.9960
VALE3	4.1	0.9920	4.3	0.9859
ITSA4	4.9	0.9917	4.2	0.9898
USIM5	4.2	0.9918	4.3	0.9915
UBBR11	4.0	0.9926	3.7	0.9932
BBAS3	3.8	0.9888	4.2	0.9881
PETR3	4.1	0.9890	4.0	0.9960
CMIG4	4.4	0.9874	4.4	0.9871
ALLL11	4.6	0.9899	4.6	0.9827
SDIA4	5.0	0.9972	5.1	0.9972
CYRE3	4.6	0.9780	4.4	0.9808
TCSL4	4.6	0.9823	4.3	0.9787
NETC4	3.2	0.7510	2.6	0.7216
LAME4	5.1	0.9664	5.4	0.9825
TNLP4	4.3	0.9885	3.7	0.9910

Source: made by the authors

Figure 4.3: CDF of the volume at the ask for PETR4 in log-log scale.

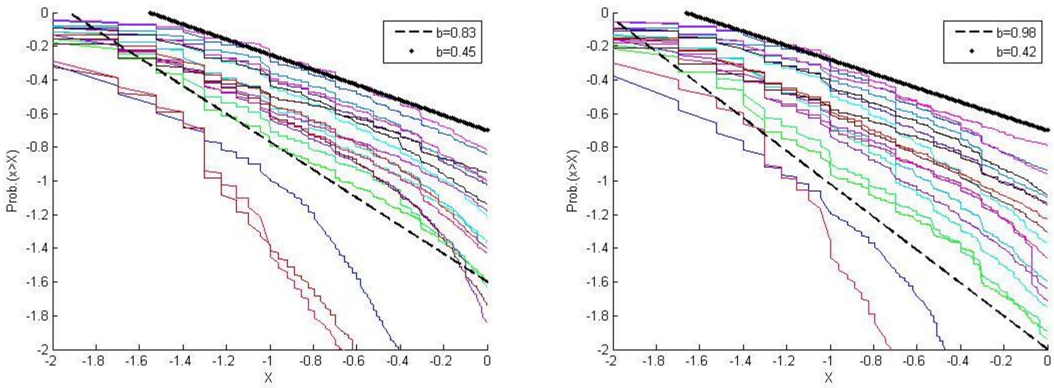


Source: made by the authors

4.1.3 First gap

Another component of the order book which has a direct impact on returns is the first gap. Once negotiated all the volume at the best price, the price shift will be proportional to the first gap. Studying a sample of 16 stocks of LSE, Farmer et al (2004) found that the distribution of the first gap follows a power law with exponent around 2.5. This statistical regularity is also observed in most of the stocks of our sample, but with a much smaller exponent. The exponents vary from 0.4 to 1 and, in general, are greater for more liquid stocks (Figure 4.4).

Figure 4.4: CDF of the first gap on the ask (left) and bid (right) side in log-log scale.

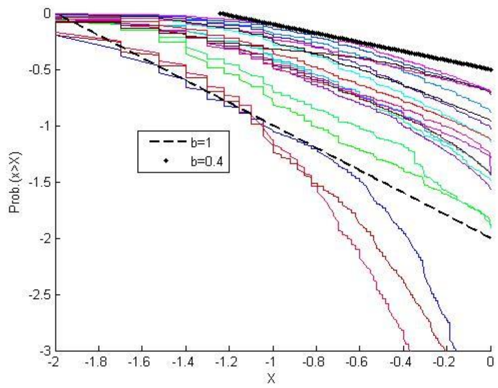


Source: made by the authors

4.1.4 Spread

A power law decay of the distribution of spread is reported by empirical studies. While Plerou et al (2005) found an exponent around 3 for a sample of 116 stocks, Mike and Farmer (2008) found an exponent spanning from 2.5 to 4.1. Figure 4.5 shows that most of the analyzed stocks have an exponent between 0.4 and 1.

Figure 4.5: CDF of the spread in log-log scale.



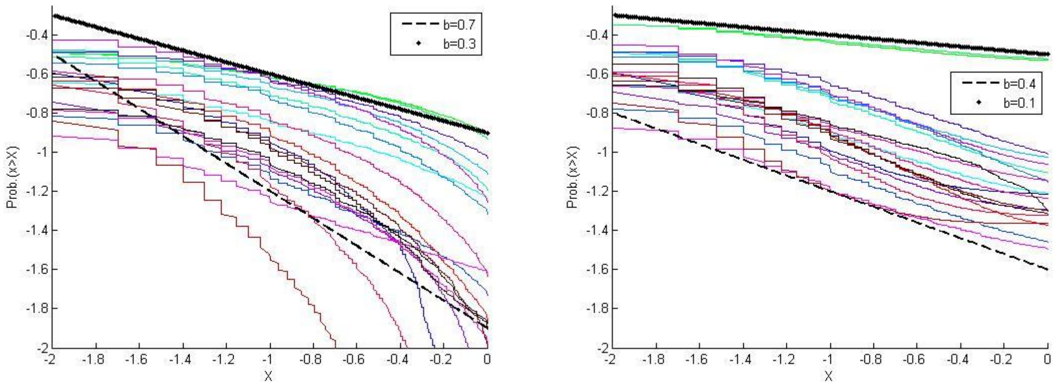
Source: made by the authors

4.2 Order flow

4.2.1 Delta of limit orders

The delta of incoming limit orders, according to some studies, has a distribution with power law decay. The estimated exponent fluctuates widely, according to the stock analyzed, the period etc. Figure 4.6 shows the cumulative distribution function for the 20 stocks of the sample. A power law decay is clearer in the case of buy orders. The exponent of limit buy orders for the two more negotiated stocks (PETR4 and VALE5) is equal to 0.1, ranging between 0.3 and 0.4 for the others.

Figure 4.6: CDF of the delta of sell (left) and buy (right) orders in log-log scale.

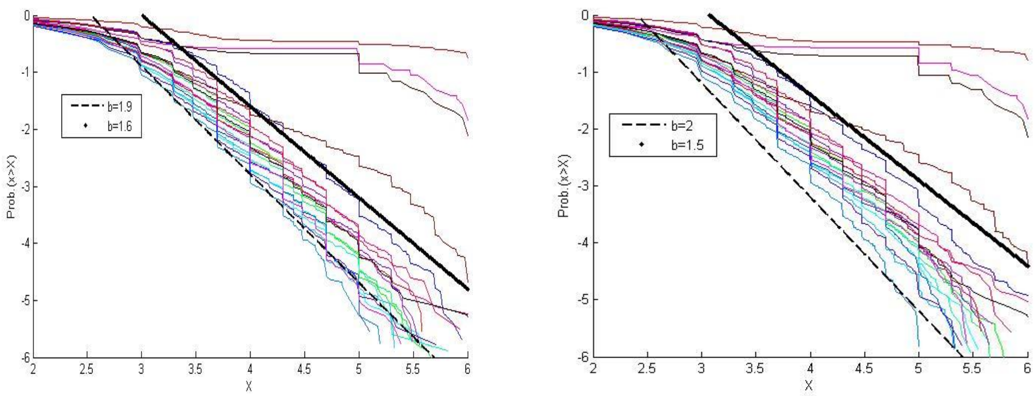


Source: made by the authors

4.2.2 Volume of orders

The empirical results on the distribution of order volume are varied. Bouchaud et al (2002), analyzing three stocks of Paris Stock Exchange, found that it follows a uniform distribution, with values spanning from 10 and 50,000. Other studies, revised in Chakraborti et al (2011), attest that the order volume obeys a power law distribution, with an exponent between 2 and 2.7. In Figure 4.7, it can be observed that the distribution of order volume of the stocks of the sample fits a power law, with exponents varying between 1.5 and 2.

Figure 4.7: CDF of the volume of sell (left) and buy (right) orders in log-log scale.



Source: made by the authors

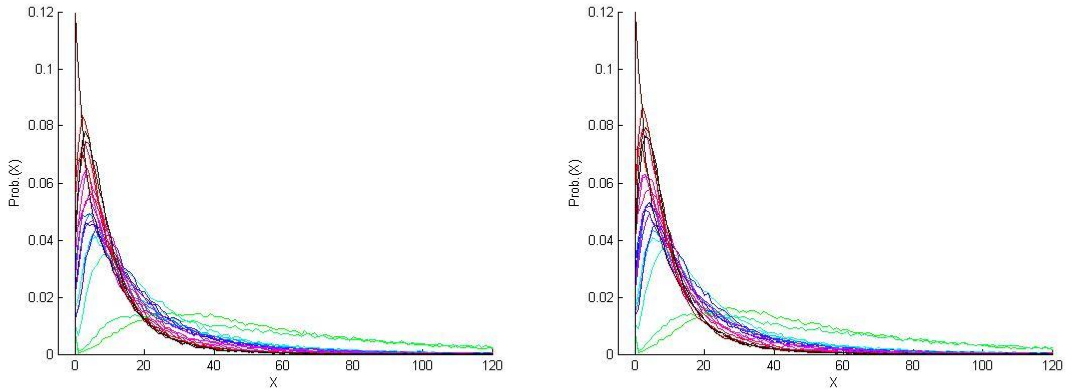
4.2.3 Arrival and lifetime of orders

The hypothesis that the arrival and withdraw of orders follow a Poisson (i.e., exponential) process is empirically tested – and refuted – by several studies. Chakraborti et al (2011) concluded that the inter-arrival time of market orders on the stock BNP Paribas is poorly fitted by an exponential distribution, being much better represented by a Weibull distribution. The same authors computed the number of transactions in a given period and checked that the log-normal and gamma distribution are good candidates to describe the empirical data, but neither really does it. Dremin and Leonidov (2005) reached a similar conclusion for Russian stocks.

The distribution of the lifetime of orders are reported to fit a power law decay (Chakraborti et al, 2011; Mike and Farmer, 2008; Challet and Stinchcombe, 2001).

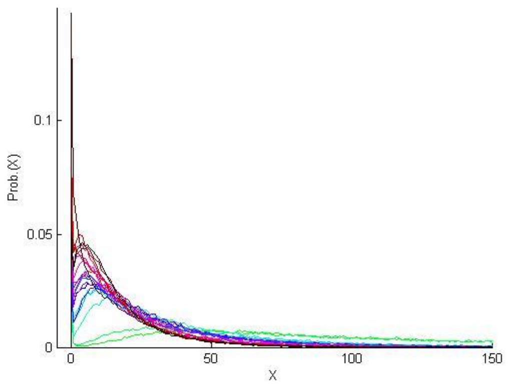
We analyzed the number of buy orders, sell orders and transactions in a time period of 5 minutes. We found that the distributions of these statistics are very similar (Figures 4.8 and 4.9) and fit quite satisfactorily a log-normal distribution. Figure 4.10 shows the fit of empirical data of PETR4 to the log-normal distribution.

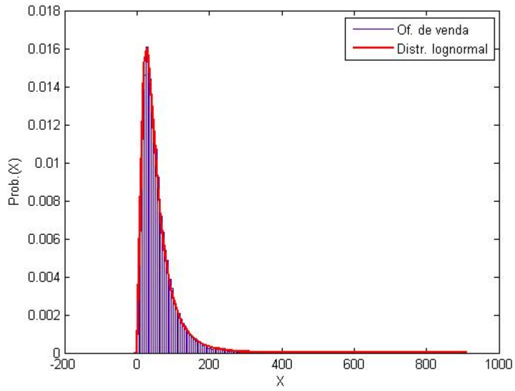
Figure 4.8: PDF of the number of buy (L) and sell (R) orders within 5 minutes.



Source: made by the authors

Figure 4.9 (left): PDF of the number of transactions within 5 minutes; Figure 4.10, right: PDF of the number of sell orders of PETR4 within 5 minutes.

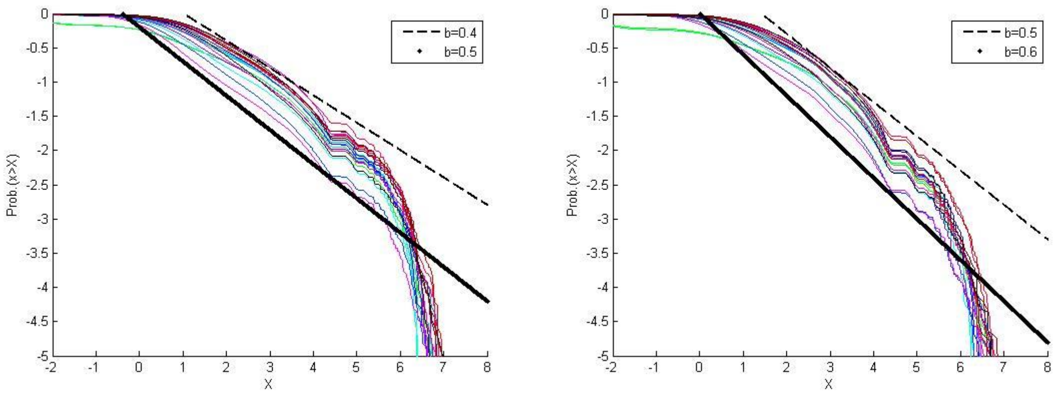




Source: made by the authors

Computing the distribution of the lifetime of limit orders, we could check that, for a range of values between 100s and 10,000s, there is a power law decay with exponent around 0.4 for sell orders and 0.5 for buy orders (Figure 4.11).

Figure 4.11: CDF of the lifetime of sell (left) and buy (right) limit orders in log-log scale.

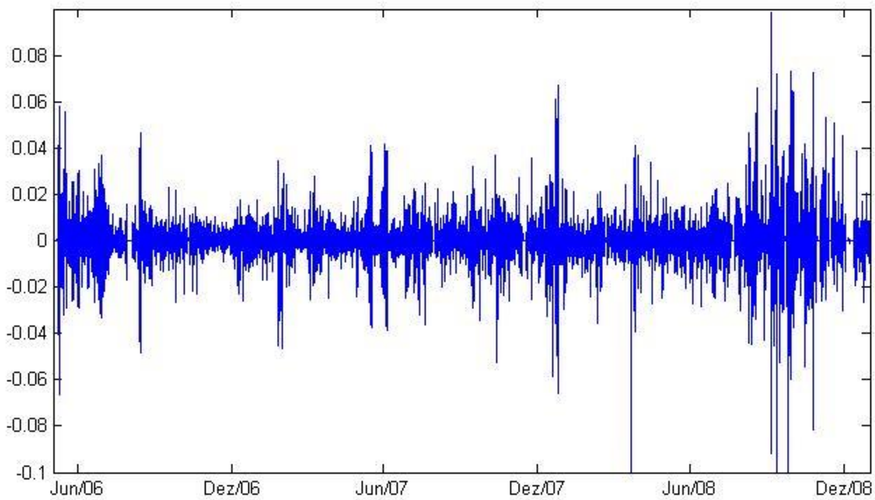


Source: made by the authors

4.3 Returns

Here, we adopted the midpoint as a proxy for the price of the stock. The return over a period of time τ $r_\tau(t)$ is equal to $\log(m(t)) - \log(m(t-\tau))$. Figure 4.12 plots the return for τ = 1 minute of PETR4. It is possible to observe some agglomeration of the volatility and a broadening of price shift at the end of 2008, reflecting the impact of the economic crises.

Figure 4.12: Returns of PETR4, τ = 1 minute.



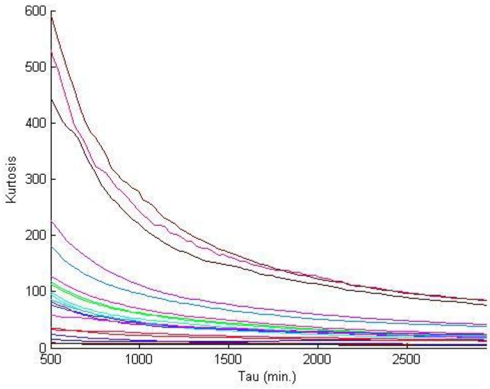
Source: made by the authors

There are the four stylized facts widely accepted for prices of stocks (Chakraborti et al, 2011): fat-tailed distribution of returns, absence of autocorrelation between returns, volatility clustering and aggregational normality.

In Figure 4.13, two of these stylized facts can be noted for the stocks of our sample. First, the kurtosis is positive, a sign of fat-tailed distribution of returns. Second, as far as τ is increased, the kurtosis approaches zero, equivalent to a normal distribution kurtosis. It reveals the data obey the aggregational normality.

REDECA – Revista Eletrônica do Departamento de Ciências Contábeis &Departamento de Atuária e Métodos Quantitativos da FEA-PUC/SP

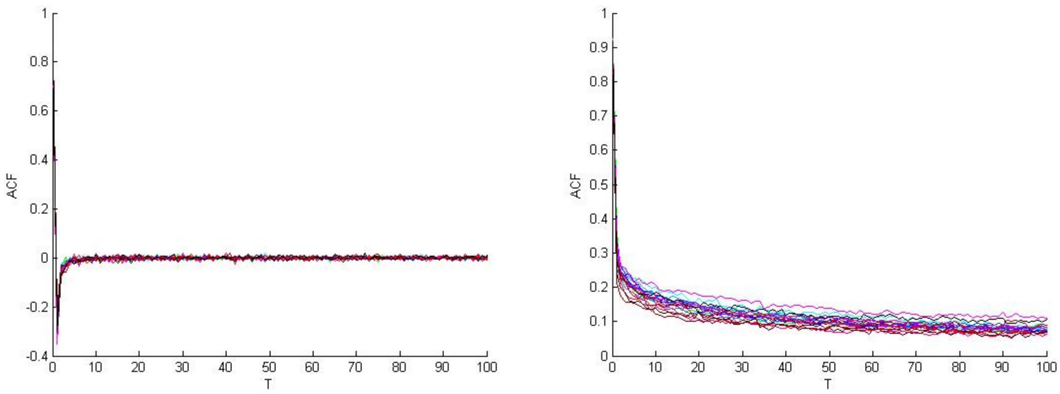
Figure 4.13: Kurtosis of analyzed stocks.



Source: made by the authors

The autocorrelation of log-returns is defined as $\rho(T) \sim \langle r_\tau(t+T) r_\tau(t) \rangle$. Autocorrelation of returns rapidly goes to zero; nonetheless, the absolute returns remains correlated even for large values of the lag T (Figure 4.13). This phenomenon is known as volatility clustering, defined by Mandelbrot (1963) in the following terms: ''large changes tend to be followed by large change – of either sign – and small changes tend to be followed by small changes''.

Figure 4.13: Autocorrelation function of signed (left) and absolute (right) returns.



Source: made by the authors

5 Market impact function

In this section, we estimate a market impact function (MIF) for the stocks of our sample. MIF measures the impact of traded volume in returns. Some studies (e.g., Farmer and Lillo, 2004; Farmer et al, 2005) examined the impact of the traded volume of an individual transaction in price shift and all of them have found a concave MIF of the form

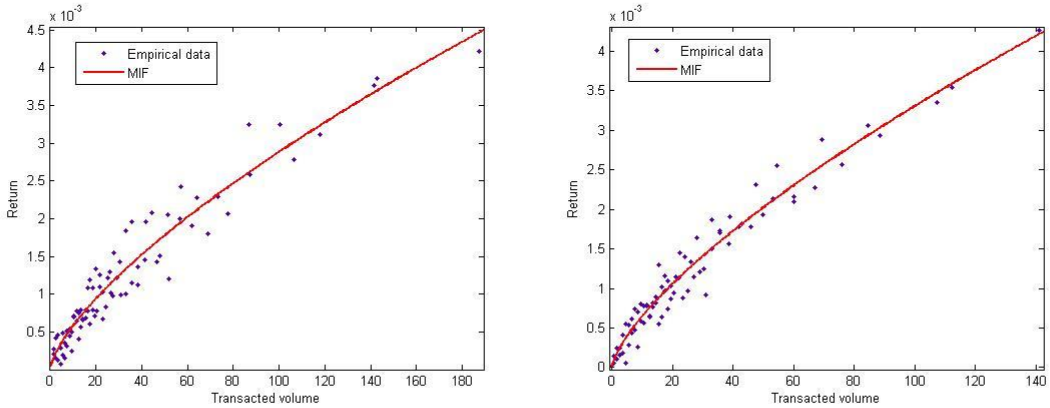
$$E[r|v] \propto \varepsilon v^{\psi} \quad (5.1),$$

were r is the return, v the transacted volume and ψ a parameter ranging from 0.2 to 0.5. On the other hand, studies of the market impact of aggregated volume of many transactions (e.g., Plerou et al, 2002; Gabaix et al, 2003, 2006) have reached variable results, concerning the value of ψ as well as the functional form of the MIF[4].

In our estimates, aggregated data were used. We work with τ = 5 minutes. Following Evans and Lions (2002), we use as a proxy for signed transacted volume $v_\tau(t)$ the number of transactions initiated by buyers minus the number of transactions initiated by sellers between t and t – τ. Then it was created the vector $W = \{0,\ w(1),\ \ldots,\ w(100)\}$, where $w(i)$ is the i-th percentile of $v_\tau(t)$. It was calculated $r_M(i)$, the average return for the values of $v_\tau(t)$ between $w(i)$ and $w(i-1)$ for i = 1, …, 100. Finally, it was calculated $v_M(i) = \frac{w(i)+w(i-1)}{2}$.

The cases in which $r_M(i)$ and $v_M(i)$ had different signs were excluded and then both were considered in terms of their absolute values. The equation 5.1 was fitted to the empirical data and Table 5.1 shows the results. The estimated ψ ranges between 0.2 and 0.7, but for all the values bellow 0.3 the fit proved to be very poor. The two more liquid stocks (PETR4 and VALE5) presented the highest value of ψ and a very good fit. The fit of empirical data to the MIF is reported in Figure 5.1 for both stocks.

Figure 5.1: Empirical data and MIF for PETR4 (left) and VALE5 (right).



Source: made by the authors

6 Conclusions

In this study, we made an extensive analysis of the microstructure of São Paulo Stock Exchange. Differently from most of similar studies, which concentrates in few liquid stocks, we have studied a big number of stocks. We have found some stylized facts observed in other stocks. The distributions of the first gap, the spread, the delta of limit orders and the volume of orders obey a power law decay. The arrival and withdraw of orders do not follow a Poisson process. Finally, the return engendered by the order book exhibits the usual statistical regularities observed at the aggregate level: fat-tailed distribution, aggregational normality, absence of autocorrelation and volatility clustering.

On the other hand, some empirical regularities reported by other studies do not appear to be universal here. For some stocks, the maximum of the average order book is observed at the best quote. The concave function is a poor representation of the market impact for most of the stocks analyzed in this study. We have found a new representation for the distribution of

[4]See Bouchaud et al (2009), Sections 5 and 6, for an extensive review on market impact.

the volume at the best price, rather than the gamma distribution of other studies. It is worth mentioning that, when a power law is found, the decaying is always smaller in the case of Brazilian stocks. It demonstrates a greater dispersion of the agents strategies and goes in hand with the greater volatility observed in emerging financial markets at the aggregate level.

Therefore, we have provide additional support to the hypothesis that there are differences between developed and emerging financial markets at the micro level, presenting data regarding the Brazilian order book. Further research should goes deep in the empirical analysis, as well as the development of theoretical models able to concatenate financial volatility at different levels.

References

Biais, B., Hillion, P., & Spatt, C. (1995). An empirical analysis of the limit order book and the order flow in the Paris Bourse. *The Journal of Finance*, 50 (5), 1655–1689.

Bouchaud, J-P., Mézard, M., & Potters, M. (2002). Statistical properties of stock order books: empirical results and models. *Quantitative Finance*, 2 (4), 251-256.

Bouchaud, J-P., J. D. Farmer e F. Lillo (2009). How markets slowly digest changes in supply and demand. In T. Hens, & K. Schenk-Hoppe (Eds.), *Handbook of Financial Markets: Dynamics and Evolution* (pp. 57-160), Amsterdam: Elsevier.

Canuto, O., & Cavallari, M. (2013). Monetary Policy and Macroprudential Regulation: Whiter Emerging Markets. *The World Bank PREM Policy Research Working Paper 6310*.

Chakraborti, A., Toke, I. M., Patriarca, M., & Abergel, F. (2011). Econophysics review: I. Empirical facts. *Quantitative Finance*, 11 (7), 991-1012.

Challet, D., & Stinchcombe, R. (2001). Analyzing and modeling 1+1d markets. *Physica A*, 300, 285-299.

Dremin, I.M., & Leonidov, A. V. (2005). On distribution of number of trades in different time windows in the stock market. *Physica A*, 353, 388-402.

Evans, R. D. D., & Lyons, R. K. (2002). Order flow and exchange rate dynamics. *Journal of Political Economy*, 110 (1), 170-180.

Farmer, J. D., Gillemot, L., Lillo, F., Mike, S., & Sen, A. (2004). What really causes large price changes? *Quantitative Finance*, 4 (4), 383-397.

Farmer, J. D., & Lillo, F. (2004). On the origin of power laws in financial markets. *Quantitative Finance*, 4 (1), 7-10.

Farmer, J. D., Patelli, P., & Zovko, I (2005). The predictive power of zero intelligence in financial markets. *Proceedings of the National Academy of Sciences of the United States of America*, 102(6), 2254-2259.

Gabaix, X., Gopikrishnan, P., Plerou, V., & Stanley, H. E. (2003). A theory of power-law distributions in financial market fluctuations. *Nature*, 423, 267-270.

Gabaix, X., Gopikrishnan, P., Plerou, V., & Stanley, H. E. (2006). Institutional investors and stock market volatility. *Quarterly Journal of Economics*, 121 (2), 461-504.

Gu, G. F., Chen, W., & Zhou, W. X. (2008). Empirical distributions of Chinese stock returns at different microscopic timescales. *Physica A*, 387 (2-3), 495-502.

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4), 394-419.

Mike, S., & Farmer, J. D. (2008). An empirical behavioral model of liquidity and volatility. *Journal of Economic Dynamics & Control*, 32, 200–234.

Plerou, V., Gopikrishnan, P., Gabaix, X., & Stanley, H.E. (2002). Quantifying stock price response to demand fluctuations. *Physical Review E*, 66(2), 027104.

Plerou, V., Gopikrishnan, P., & Stanley, H.E. (2005). Quantifying fluctuations in market liquidity: Analysis of the bid-ask spread. *Physical Review E*, 71, 046131.

Zaccaria, A., Cristelli, M., Alfi, V., Ciulla, F., & Pietronero, L. (2010). Asymmetric statistics of order books: The role of discreteness and evidence for strategic order placement. *Physical Review E*, 81, 066101.

REDECA – Revista Eletrônica do Departamento de Ciências Contábeis &Departamento de Atuária e Métodos Quantitativos da FEA-PUC/SP