



An Innovative Thinking on the Concepts of Ex-Ante Value, Ex-Post Value and the Realized Value (Price)

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Abstract: Researchers have been considering the realized value as the ex-post realization of the ex-ante value. They have argued that that the realized values have failed to estimate the expected value in asset-pricing models. We provide a new definition of the ex-post measurement and we show that considering realized value as the ex-post realization of the ex-ante value is misleading and this has led to the failure in estimating the expected value.

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INTRODUCTION

In theory, researchers can establish risk-return relationship; empirically, the unobservable nature of the *ex-ante* expected return hinders the estimation of the relationship between the risk and return. In general, it is believed that investors know their expected value and the variance-covariance matrix, and based on these, they form the price of an asset in the market. Thus in the text books, researchers have been using realized returns assuming that realized returns are normally distributed with mean μ_i and a variance of σ_i^2 ; $R_{it} \sim N(\mu_i, \sigma_i^2)$, and over the sample period, the average of these realized returns will match the *ex-ante* expected return. This implicit belief has led the researchers to assume realized returns as a sample of returns in estimating the expected return, i.e., they consider realized value as the *ex-post* realization of the *ex-ante* expectations.

We argue that this belief on the convergence of realized return on *ex-post* return is misleading. The disparity between *ex-post* realization and *ex-ante* prediction is well addressed by Sharpe (1978); and Campello *et al.* (2008) also believe that the distribution of expected returns does differ from the distribution of realized (*ex-post*) returns. Elton (1999) concluded that the realized (*ex-post*) return is a poor proxy for the expected return because of its deficiency in reflecting the nature of *ex-ante* expectations. None of the researchers have identified the reason behind this disparity between these two values, however. From the point of view of asset-pricing model, we introduce new definition of the *ex-post* value and show clear distinction between the realized value (price) and the *ex-post* value, and in turn, it proves that realized price cannot be the *ex-post* realization of the *ex-ante* values. *How the ex-post value and the realized value differ from each other?*

THE EX-ANTE RETURN, EX-POST RETURN AND THE REALIZED RETURNS

The main focus of the asset-pricing model is to explain the risk-return relationship. Theoretically, we can establish risk-return relationship (for example, CAPM). However, unobservable nature of the *ex-ante* expected return hinders estimating the empirical risk-return relationship. As a result, in empirical analysis, most of the researchers consider realized return as the *ex-post* realization of the (*ex-ante*) return, i.e., they assume realized return as a sample of return. For example, they assume that (*ex-ante*) return¹ are normally distributed with mean μ_i and variance of σ_i^2 , $\tilde{R}_{i,t} \sim N(\mu_i, \sigma_i^2)$. They have used the average realized return and sample

¹ In general, expected return has been considered as the '*ex-ante* return' by the researchers. As we have discussed later in the paper that the *ex-ante* literally means the random future values. If we define *ex-ante* return as the expected return, we are disregarding the randomness of the future values. Therefore, we have defied the returns as '*ex-ante* returns' in this paper instead of the 'expected returns' as has been considered by the other researchers.

variance as estimators of the *ex-ante* expected return and the *ex-ante* variance. Nevertheless results of the empirical analysis were almost inconclusive.

Some researchers intuitively believe that the realized return cannot be the *ex-post* realization of the (*ex-ante*) return and consequently empirical estimation differs from the *ex-ante* expectation. In this section we depict the inability of the realized return as the *ex-post* realization of the *ex-ante* and present that *ex-post* value is different from the realized value. We portray our argument from the pricing point of view and in doing so we show that the information set in the price is different from the information set in the *ex-post* value. Our argument is based on the following simplified assumptions:

- (i). In an one-period setting, price is the discounted value of the next period's expected price, $p_{i,t} = \frac{E(\tilde{p}_{i,t+1})}{d_i}$ where $d_i > 1$. In addition, we assume that $E(\tilde{p}_{i,t+1})$ incorporates all future information available at t .
- (ii). The state of future economy changes with time.

Assumption (i) states that for any risky asset the investors are assumed to expect positive payoffs in future and can be considered as one of the basic assumptions in valuation. Assumption (ii) can be considered as the base of our argument. Most of the researchers assume a steady state of the economy where there is no change in the fundamental economic variables. Rather they consider any change in the information set (surprises) as a change in variables other than the fundamentals. And for a sample, these surprises are expected to be cancelled out. We assume that any change in the economy is a result of the changes in the economic variables, both fundamentals as well as firm specific ones. This may lead us to assume that investors' forecasts about the asset's expected price would increase (decrease) with forecasted positive (negative) changes in the economic variables. Besides, researchers have been using the realized return in empirical tests, and in reality the economy is changing also. Thus our second assumption is much closer to the reality.

Most of the researchers have been using realized return in establishing the empirical risk-return relationship; we introduce 2 scenarios and argue on the inability of the realized return to explain the risk-return relationship. As we proceed, we discussed on the different information sets in the asset-pricing, and gradually, we present the difference between the realized value and the *ex-post* value. We conclude that realized return cannot be a sample of return.

An example:

We begin with a simple example for better understanding of our argument. We show that when the assumptions (i) and (ii) hold, average realized returns cannot estimate the expected return. Investor's expected price would rise (fall) with the favorable (unfavorable) future economic forecasts. We start our argument with a series of unfavorable future economy in scenario 1. Under one-period model settings, we assume that price in every period is formed

based on the expected price of the next period. Let us assume the expected prices for $(t + 1)$ to $(t + 4)$ at t , $(t + 1)$, $(t + 2)$ and $(t + 3)$ are 105, 95, 89 and 83 respectively. If we assume 5% expected return² for the investors, we would get the price for t to $(t + 3)$ as $(105/1.05)$, $(95/1.05)$, $(89/1.05)$ and $(83/1.05)$ respectively. For this series the average realized return would be negative. Note that our expected return is 5% in scenario 1. The sample average realized return for these types of series cannot estimate the expected return of 5%. Why average realized return fails to estimate the expected return?

Scenario 1: Realized return and the risk-return relationship in downward Market

This table forecasts the future values from $(t+1)$ to $(t+4)$ in a down-ward market. The expected return (cost of capital) is 5% (i.e., discount rate, $d_i = 1.05$). For simplicity of the argument we assume expected return as constant.

	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$
$E(\tilde{p}_{t+\tau})$		105	95	89	83
$p_{t+\tau}$	$105/1.05$	$95/1.05$	$89/1.05$	$83/1.05$...
$r_{t+\tau+1} = \frac{p_{t+\tau+1}}{p_{t+\tau}}$		0.905	0.937	0.933	...

For $(t + 1)$ in scenario 1, researchers would consider $(95/1.05)$ as the *ex-post* realization of *ex-ante* price for t , i.e., $(95/1.05)$ is treated as a realized value of the *ex-ante* distribution of future price of $\tilde{p}_{i,t+1}$ for $(t + 1)$ at t . Can $(95/1.05)$ at $(t + 1)$ be a *ex-post* value of the future price of $(t + 1)$ for t ?

The price at $(t + 1)$ is the discounted expected price of $(t + 2)$. In this example, the price $(95/1.05)$ at $(t + 1)$ is derived from the information on the future price for $(t + 2)$ which is available at $(t + 1)$. In general, the *ex-post* value at $(t + 1)$ is the observed value from the information on t to $(t + 1)$. $(95/1.05)$ cannot be the *ex-post* value at $(t + 1)$ as this value is derived from the information of $(t + 2)$ instead of the information set of t to $(t + 1)$. Under assumption (ii), the expected price of $(t + 2)$, $E(\tilde{p}_{i,t+2})$, has no relation to the distribution of $\tilde{p}_{i,t+1}$ at $(t + 1)$. So the realized return can neither be the *ex-post* return nor the sample of return.

² Although the expected rate of return (the discount rate) might change with the changes in the economic forecasts, for simplicity, we consider constant discount rate in this paper. Note that, the argument of this paper can support a model with changing discount rate scenario also.

In scenario 2, with favorable economic forecasts, the expected values increase from 105 in $(t + 1)$ to 150 in $(t + 4)$. We can consider scenario 2 as an illustration of the Japanese bubble during 1985-90. With this increase, the prices also increase from 100 at t to 143 in $(t + 3)$. The average realized return for this type of upward series will be much greater than the expected return of the asset (5% in this case). Besides, as we have argued before, $(120/1.05)$ cannot be considered as the *ex-post* value at $(t + 1)$ because $(120/1.05)$ is derived from the information set on the expected price of $(t + 2)$ available at $(t + 1)$.

Scenario 2: Realized return and the risk-return relationship in upward Market

This table forecasts the future values from $(t+1)$ to $(t+4)$ in an upward market. The expected return (cost of capital) is 5% (i.e., discount rate, $d_i = 1.05$). For simplicity of the argument we assume expected return as constant.

	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$
$E(\tilde{p}_{i,t+\tau})$		105	120	135	150
$p_{i,t+\tau}$	$105/1.05$	$120/1.05$	$135/1.05$	$150/1.05$...
$r_{i,t+\tau+1}$ $= \frac{p_{i,t+\tau+1}}{p_{i,t+\tau}}$		1.143	1.125	1.111	...

None of the researchers have argued on the information sets in the price as well as in the *ex-post* return. In this section, with simple illustrative examples under assumption (i) and (ii), we have shown that the information sets in price and in *ex-post* return are different, and price cannot be considered as the *ex-post* realization of the *ex-ante* expectation.

REALIZED RETURN AND THE EX-POST RETURN

In this section, we provide a general discussion on the difference between realized return and *ex-post* return. We have divided information at t into two parts for better understanding, and we define information as:

$$\Phi_t = \Phi_t^H + \Phi_t^{F_{t+1}} \quad (1)$$

where, Φ_t is the total information set available at t , Φ_t^H is the past information set on $(t - 1)$ to t available at t , and $\Phi_t^{F_{t+1}}$ is the future information set on $(t + 1)$ that is incorporated at t . Past information set is assumed to be comprised of the results of the operating activities between $(t - 1)$ to t . In contrast, the economic information as well as the firm's future policies is

incorporated in the future information set. Under assumption (i), price p_{it} is the discounted value of $E(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$. Similarly, price $p_{i,t+1}$, is the discounted value of $E(\tilde{p}_{i,t+2}|\Phi_{t+1}^{F_{t+2}})$. At $(t + 1)$, $p_{i,t+1}$ does not incorporate past information set $\Phi_t^{H_t}$; ³ it is derived from the future information set of $\Phi_{t+1}^{F_{t+2}}$. The figure 1 explains the difference between information sets in price and the *ex-post* value.

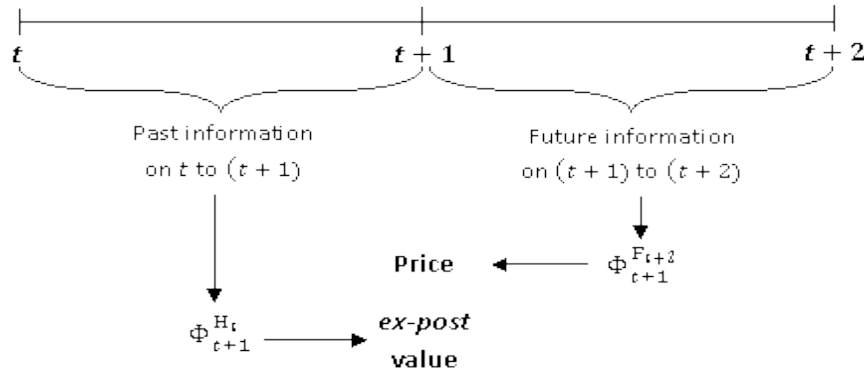


Figure 1: Price and *ex-post* value

In the following discussion, we provide further explanation to confirm that the realized return cannot be the *ex-post* return. We define (*ex-ante*) return at time t , $\tilde{r}_{i,t+1}$ as,

$$\tilde{r}_{i,t+1} = \frac{\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}}}{p_{i,t}|\Phi_t^{F_{t+1}}} \quad (2)$$

and, the realized return, $r_{i,t+1}$, is defined as,

$$r_{i,t+1} = \frac{p_{i,t+1}|\Phi_{t+1}^{F_{t+2}}}{p_{i,t}|\Phi_t^{F_{t+1}}} \quad (3)$$

The researchers consider $(p_{i,t+1}|\Phi_{t+1}^{F_{t+2}})$ as the *ex-post* realization of $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$, i.e.; they have been assuming $(p_{i,t+1}|\Phi_{t+1}^{F_{t+2}})$ as the sample of the distribution of future random price of $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$. The return at t in equation (4.2) incorporates the information about the time period $(t + 1)$, available at t . In equation (3), $(p_{i,t+1}|\Phi_{t+1}^{F_{t+2}})$ has no relation with $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$ under assumption (ii), however. Instead, $(p_{i,t+1}|\Phi_{t+1}^{F_{t+2}})$ is the discounted expected value of $(\tilde{p}_{i,t+2}|\Phi_{t+1}^{F_{t+2}})$. The realized return of $(t+1)$ in equation (3), includes information about periods

³ Past information, for example as cited by Elton (1999), high earnings announcements of MacDonald, has little or no role in forming future expectation of the investors. *Does high earnings announcement really lead to higher future price?* In TSE, the annual earnings for Nintendo was the highest in March of 2009 at JPY 279 billion (approx); the price dropped from JPY 71,900 in 2007:10 to JPY 23,180 in 2009:10 following the earnings information, however. If positive (negative) past information has an impact on the following price, the price would have increased (decreased) following the information. The drift in Nintendo’s price, even with the highest earnings information, can be an example of the absence of the effect of past information on the price.

($t+1$) and ($t+2$) for $(p_{i,t}|\Phi_t^{F_{t+1}})$ and $(p_{i,t+1}|\Phi_{t+1}^{F_{t+2}})$ respectively. The information sets in $\tilde{r}_{i,t+1}$ and $r_{i,t+1}$ are different. These two values are derived from different information sets of different time periods. As a result realized return can neither be *ex-post* return nor a sample of return.

ALTERNATIVE DEFINITION OF EX-POST VALUE

In section 3, we have shown that the present believe on the *ex-post* return is misleading. How can we measure the *ex-post* return? At t , we consider, the *ex-ante* prediction follows⁴,

$$\tilde{p}_{i,t+1} = (p_{i,t}|\Phi_t^{F_{t+1}}) + (\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}}) \quad (4)$$

where, $(\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}})$ is random operating value for t to $(t + 1)$ based on available information set $\Phi_t^{F_{t+1}}$ at t . We assume earnings, $\tilde{x}_{i,t+1}$, as the random operational outcome from t to $(t + 1)$ realized at $(t + 1)$. We observe earnings for t to $(t + 1)$, i.e.; $(x_{i,t+1}^*|\Phi_{t+1}^{H_{t+1}})$. Thus, we define the *ex-post* value at ($t+1$), $(v_{i,t+1}^*|\Phi_{t+1}^{H_t})$ as,

$$v_{i,t+1}^* = (p_{i,t}|\Phi_t^{F_{t+1}}) + (x_{i,t+1}^*|\Phi_{t+1}^{H_{t+1}}) \quad (5)$$

where, $x_{i,t+1}^*$ is the observed earnings at $(t + 1)$. The value in equation (5) is the realized value of *ex-ante* random price of $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$ for $(t + 1)$ made at t . The realized price, $p_{i,t+1}$, is not the *ex-post* realization of $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$; whereas, $x_{i,t+1}^*$ is the observed earnings for t to $(t + 1)$ at $(t + 1)$. The *ex-post* return, $r_{i,t+1}^*$, can be written as:

$$r_{i,t+1}^* = \frac{v_{i,t+1}^*|\Phi_{t+1}^{H_{t+1}}}{(p_{i,t}|\Phi_t^{F_{t+1}})} \quad (6)$$

⁴ Readers might question that: How the investors will forecast the future when the future looks gloomy and next period's expected value is thought to be negative? that is, when $\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}} < 0$, how the investors will make their future forecasts?

Following equation (4.4), when $\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}} < 0$, investors will consider another variable $\tilde{\theta}_{i,t+1}|\Phi_t^{F_{t+2}}$ which includes the information on the periods $(t + 2)$ onwards.

$$\tilde{p}_{i,t+1} = (p_{i,t}|\Phi_t^{F_{t+1}}) - (\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}}) + (\tilde{\theta}_{i,t+1}|\Phi_t^{F_{t+2}})$$

The idea behind this is that, even though the next period's values are negative, the following periods information makes the forecasts positive in the sense that:

$$-(\tilde{x}_{i,t+1}|\Phi_t^{F_{t+1}}) + (\tilde{\theta}_{i,t+1}|\Phi_t^{F_{t+2}}) > 0$$

In the later section we conduct an empirical test on the *ex-post* measure of the above relationship. And we have provided proof supporting our assumption.

As a concluding remark of section 4, the *ex-ante* value at t is the expected value of $E(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$ for $(t+1)$. $E(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$ is discounted to derive $p_{i,t}$ at t . The *ex-post* value at $(t+1)$ is the realized (observed) value of time t 's anticipation of $(\tilde{p}_{i,t+1}|\Phi_t^{F_{t+1}})$ that we would observe as we move to $(t+1)$. In contrast, price $p_{i,t+1}$ is derived from $E(\tilde{p}_{i,t+2}|\Phi_{t+1}^{F_{t+2}})$ at $(t+1)$. In section 3 we have argued that the information sets in these values are different. At $(t+1)$, price $p_{i,t+1}$ incorporates the information set $\Phi_{t+1}^{F_{t+2}}$ on $(t+2)$, whereas the *ex-post* value at $(t+1)$, $v_{i,t+1}^*|\Phi_{t+1}^{H_{t+1}}$, is observed from the operational activities of t for $(t+1)$.

A NUMERICAL EXAMPLE ON THE EX-ANTE RETURN, EX-POST RETURN AND THE REALIZED RETURN

The Latin word '*ex-ante*' means 'beforehand'. In models where there is uncertainty, that is resolved during the course of events, the *ex-ante* values are those that are calculated in advance of the resolution of uncertainty. In finance, for example, *ex-ante* is the future random values. As a numerical example, let $E_t(\tilde{p}_{i,t+1})$ and R_i be \$110 and 10% respectively. The price at t , $p_{i,t}$, would be \$100. Now, as we move to $(t+1)$, the price $p_{i,t+1}$ will be the discounted expected value of $(t+2)$, i.e., $E_{t+1}(\tilde{p}_{i,t+2})$. If for some reasons, at $(t+1)$ the investors predict a macro-economic downturn in $(t+2)$. At $(t+1)$ investors predict $E_{t+1}(\tilde{p}_{i,t+2})$ to be \$99.⁵ Thus the price at $(t+1)$, $p_{i,t+1}$, will be \$90. This downward movement of the price is the result of the unfavourable forecast of $E_{t+1}(\tilde{p}_{i,t+2}|\Phi_{t+2})$. Under these circumstances, the *ex-ante* returns for both t and $(t+1)$ are 10% whereas the realized return at $(t+1)$ is -10%. The price \$90 at $(t+1)$ is not observed from $\tilde{p}_{i,t+1}$, rather $p_{i,t+1}$ is the discounted value of $E_{t+1}(\tilde{p}_{i,t+2})$. Thus, realized value (or the price) cannot be the sample of the *ex-ante* expectations. *Can realized return be the ex-post realization of the ex-ante expectations?*

Continuing with the numerical example, let the operating earnings for t to $(t+1)$, i.e.; $x_{i,t+1}^*$, be \$8. Thus the *ex-post* value $v_{i,t+1}^*$ becomes \$108 and the *ex-post* earnings is 8%. The realized return at $(t+1)$ is -10% whereas the *ex-post* return is 8%. *How rational will substituting 8% with -10% be?* In other words, we cannot substitute the *ex-post* return of 8% with the realized return of -10%. The *ex-post* earnings is positive whereas the realized earnings is negative. The realized return cannot be the *ex-post* realization of the *ex-ante* returns.

In the empirical tests of CAPM, researchers are assuming that the realized return, $r_{i,t+1}$, is a sample of return $\tilde{r}_{i,t}$, and the average of the realized return, \bar{r}_i , will be the best estimate of the expected return, $E(\tilde{r}_i)$. We have argued in this section that, the realized return cannot be a sample of return. We believe, the intuition that realized return as the *ex-post* realization of the

⁵ For simplicity, we have assumed that the variance-covariance matrix for the investor is constant and any change in the macro-economic variable will be reflected in the changing expected value.

return is misleading. Redefining the *ex-post* value will portrayed the distinction between the *ex-post* value and the realized value. When the realized value is not the *ex-post* realization of the *ex-ante* value, it concludes that realized return is not the sample of return.

The term *ex-post* literally means "after the fact". For any *ex-ante* value, the *ex-post* value will be observed as we moved to that particular period and when all the uncertainty has been resolved. For example in figure 6.2, at t , the future value of asset i for $(t+1)$ can be considered as the *ex-ante* value, i.e.; $\tilde{p}_{i,t+1}$ would be the *ex-ante* value for t . In contrast, the *ex-post* value is the observed value at $(t+1)$ from $\tilde{p}_{i,t+1}$, and this value is the result of operations from t to $(t+1)$. In other words, the *ex-post* value can be defined as the observed value of $\tilde{p}_{i,t+1}$, forecasted at t , that is realized at $(t+1)$. However, the realized price at $(t+1)$, $p_{i,t+1}$, is the discounted future value of $(t+2)$. Although we get two values at $(t+1)$, the *ex-post* value and the realized price, these values are different as they are derived from different time periods.

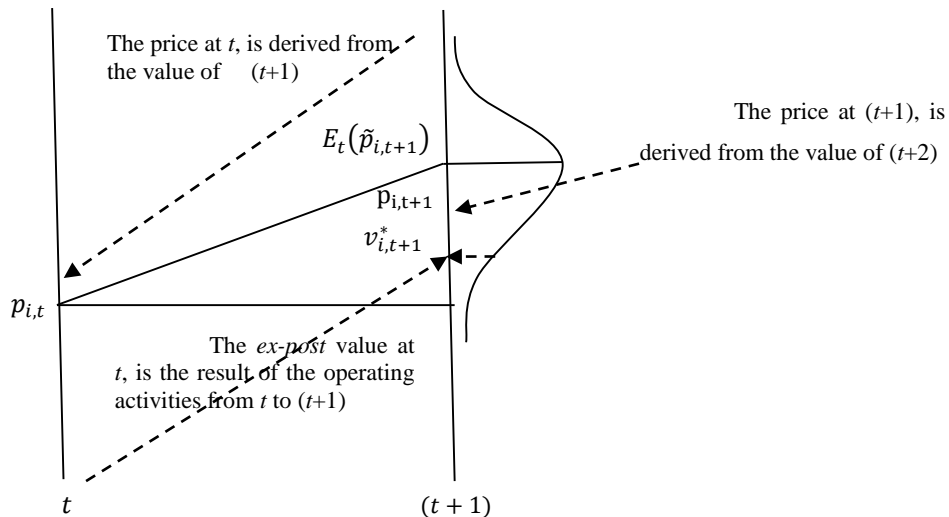


Fig 2: Derivation of Prices and *ex-post* value

We defined $x_{i,t+1}^*$ as the *ex-post* earnings of the distribution of random values of $\tilde{p}_{i,t+1}$ in $(t+1)$, which is observed as we move to $(t+1)$. The researchers believe that the *ex-post* average return provides a good estimate of the *ex-ante* expected return. As the *ex-ante* expectation is unobservable, empirical tests of CAPM assumes that the probability distribution generating the *ex-post* outcomes is stationary over time and realized return could be substituted as the sample of *ex-post* realization of the *ex-ante* expectations. *Are both of these values, the ex-post and the realized value (price), generated from the same distribution of future values? Are both $v_{i,t+1}^*$ and $p_{i,t+1}$ derived values of the distribution of $\tilde{p}_{i,t+1}$?* Figure 2 clearly explains that these values are not the same. Instead, these values provide different information to the researchers. Although these two values rarely coincide, none of the values can be treated as substituting the other value.

CONCLUSION

In this paper, we focused on the belief of considering realized return as a sample of return. Under assumptions (i) and (ii), we have shown that realized return cannot be the *ex-post* realization of the *ex-ante* expectation.

The researchers can establish the risk-return relationship in theory. The unobservable nature of the expected return has led the empirical researchers to use realized return as a sample of return. And the measurement of the empirical risk-return relationship has been inconclusive and controversial. As a result, a number of researchers have introduced new models to measure the empirical risk-return relationship.

For example, Fama and French (1992) have introduced the 3-factor model in an attempt to explain the empirical risk-return relationship. Their model gained popularity as they focused on forming an empirical model that would fit the realized return data. The model is used to explain the *ex-ante* risk-return relationship from the realized return data. We have shown that realized return can neither be the *ex-post* return nor the sample of return. What economic implication does the realized return data contain?

This thesis is the first one to explicitly define the *ex-post* value, and we have shown that realized value and the *ex-post* value are different because of the differences in the information sets. We conclude that realized return cannot be the *ex-post* realization of the (*ex-ante*) return, i.e., realized return cannot be a sample of return.

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