Abstract
With the method of duopoly game theory, the effect of firms’ innovation ability on the choice between product and process innovation is studied. The concept of cost coefficient of product innovation is introduced, and the criterion equation for the innovation type is derived. The following conclusions are made: the more the product innovation ability, the more the possibility for the firms to carry out the product innovation in both the Bertrand and the Cournot competitions. With the decrease of the product innovation ability, for the high-quality firms, Cournot competitor turns to select the process innovation earlier than the Bertrand competitor. But for the low-quality firms, the Bertrand competitor would select the process innovation firstly.

Key words: Innovation Ability, Process Innovation, Product Innovation, Competition

Introduction
Traditional view believed that the market concentration is helpful to the innovation. But some opposite opinion suggested that the intensified competition pushes forward the innovation. Delbono and Denicolo(1990)concluded that for the homogeneous products, the incentives to reduce cost in the Bertrand competition is larger than in the Cournot competition[1]. Taking into account of the product differentiation, Bester and Petrakis (1993) thought that the innovation incentive to reduce cost is lager in the Cournot competition with the larger differentiation, and is larger in the Bertrand competition with the small differentiation[2]. Bonanno,G., B.Haworth(1998) studied that for the high product quality enterprise, it selects the same type innovation, or if different selection
exists, the product innovation is selected in Bertrand and the process innovation is selected in the Cournot.

And for the low product quality, the opposite is true. Pia Weiss (2003) got the conclusion that firms tend to implement product innovation with the condition of intensified competition and lower cost, and process innovation with the weaker competition. Jan Boone (2000) divided the firms into four kinds based on the cost levels, which are complacent, eager, struggle and faint. If the competition is getting increasing, complacent firms select product innovation, eager firms select process innovation and product innovation, struggle firms select process innovation and faint firms select nothing.

We can see that many studies have been carried out on the relationship between competition and innovation. Recently the effect of competition on the product innovation and process innovation is getting more studies. But it is found that almost all the studies did not put the firm innovation ability into consideration when they studied the relationship between the competition and the innovation. In fact, we can find that in the real operation of the firms, for a certain new product, some firm’s produce it with a little invest, but others with lots of invest. Obviously, it related with the developing ability for new products. It would affect the profit obtained from the new product, then, the decision to carry out the developing action or not would be considered by the firms, and the innovation direction is sure to be influenced by the innovation ability.

In this paper, the effect of innovation ability on the choice of product innovation and process innovation is studies within duopoly market with vertical product differentiation. Based on the vertical differentiation model of Mussa and Rosen (1978), adopting the study method of game from Bonanno, G., B. Haworth (1998), introducing a concept of cost index for product innovation, a criterion equation for the judgment of the innovation type is derived and the relationship between firm innovation ability, competition, product quality and the innovation type is set up.

**Theoretical Frame**
The product innovation in this paper is referred to as the improvement of the product properties, that is to say the only vertical differentiation is considered in the study. The basic theoretical model is based on the vertical differentiation by Mussa and Rosen (1978) as well as the analyzing method by Bonanno, G., and B. Haworth (1998). With the innovation ability considered, the theoretical frame is formed.

**Vertical Differentiation By Mussa And Rosen (1978)**

There are two firms, one is $H$ with high quality product $k_H$, and another is $L$ with low quality product $k_L$. The number of consumers is $N$, and every consumer buy one unit of product at most. In the case of Bertrand competition, the equilibrium profit function is

$$
\pi^B_H(k_H, k_L, \Delta H, \Delta L) = N k_H (k_H - k_L)^2 - 2 k_H (k_H - k_L) (k_L + \Delta L)^2 \left(4k_H - k_L\right)^2 (k_H - k_L)
$$

$$
\pi^B_L(k_H, k_L, \Delta H, \Delta L) = N k_L (k_H - k_L)^2 - 2 k_L (k_H - k_L) (k_L + \Delta L)^2 \left(4k_H - k_L\right)^2 (k_H - k_L)
$$

In the case of Cournot competition, the equilibrium profit function is

$$
\pi^C_H(k_H, k_L, \Delta H, \Delta L) = N k_H (2k_H - k_L - 2 \Delta H + \Delta L)^2 \left(4k_H - k_L\right)^2
$$

$$
\pi^C_L(k_H, k_L, \Delta H, \Delta L) = N k_L (2k_H - k_L - 2 \Delta L + \Delta H)^2 \left(4k_H - k_L\right)^2
$$

$c_H, c_L$---the unit cost of product of high quality firm and the low quality firm respectively, with ($c_H > c_L$)


Assume that there is no other cost involved in the implementation of the innovation (e.g. It has hired a team of engineers). Define a product/process investment opportunity as a triple ($\Delta c, \Delta k, \alpha$) where $\alpha$ the cost of implementing the innovation is, $\Delta c$ is the expected reduction in unit cost if process innovation is pursued, $\Delta k$ is the expected quality increase if product innovation is pursued. A process/product investment opportunity ($\Delta c, \Delta k, \alpha$) is profitable if the expected increase in profit from at least one of the two types of innovation is greater than $\alpha$. Based on this analyzing logic, the following
conclusions are obtained: for the high-quality firm, given a profitable product/process investment opportunity \((\Delta c, \Delta k, \alpha)\), either both the Bertrand and the Cournot competitor choose the same type of innovation or, if they make different choice then the Bertrand competitor chooses product innovation, while the Cournot competitor chooses process innovation.

**Model Based On Innovation Ability**

Generally speaking, product innovation is related with new products and new services, and process innovation is related with the cost decrease of the existing products. Same as the model introduced above, we consider the product innovation as the improvement of the existing products qualities. The quality improvement needs the engineer team to do research work. On the other hand, it needs extra equipment, operator or high quality raw materials and so on. Guo Xiaochuan quoted the opinion that in the new commercial environment, the criterion for the product innovation is “supply the highest value at the shortest time and the lowest cost”\cite{7}. At the same time and the same value created, the lower the cost, the stronger the product innovation ability. Bi Kexin, Ding Xiaohui and Feng Junying\cite{8} posed the measurement index for the process innovation ability, among seven indexes, the basic index is the profit one, including mainly salary decrease, material and energy reduction, crap decrease as well as the labor efficiency increase. All are related with the cost reduction. So we can say that the more the cost decrease, the stronger the process innovation ability is.

**Product innovation ability**

We first consider the high-quality firm. The profit function can be expressed as

\[
\pi_H = \pi_H (k_H, k_L, c_H, c_L)
\]

(5)

Assume that the increase of product quality from \(k_u\) to \(k_u + \Delta k_u\) makes the unit cost increased by \(\Delta c_{uH}\) due to the extra means applied on the product. Then the increment of the profit can be expressed as

\[
\Delta \pi_{uH} = \pi_u(k_u + \Delta k_u, k_L, c_u + \Delta c_{uH}, c_L) - \pi_u(k_u, k_L, c_u, c_L)
\]

From the profit expression (5), we can get the cost expression:

\[
c_H = c_H (\pi_H, k_H, k_L, c_L)
\]
Then we have
\[ \Delta c_H = c_H \left( \pi_H \cdot k_H + \Delta k_H \cdot k_L \cdot c_L - \Delta c_H \cdot c_L \right) \]  
(6)

It means that with the increment of the quality \( \Delta k_H \), the unit cost also have an increment \( \Delta c_H \), which offset the profit increase by \( \Delta k_H \). We call this \( \Delta c_H \) as critical cost of product innovation, denoted with \( \Delta c_H^* \).

Generally, the profit increase due to the increase of quality is not equal to the profit decrease caused by the cost increase. We use \( \Delta c_H = \Delta c_H \alpha_{ih} \) (\( 0 \leq \alpha_{ih} \leq 1 \)) to express the product innovation cost. \( \alpha_{ih} \) is called as cost coefficient of product innovation. The less the \( \alpha_{ih} \) is, the stronger the product innovation ability of the firms.

Then the profit increment can be expressed as
\[ \Delta \pi_{ih} = \pi_H \left( k_H + \Delta k_H, k_L, c_H + \Delta c_H \alpha_{ih}, c_L \right) - \pi_H \left( k_H, k_L, c_H, c_L \right) \]

Process innovation ability
For the high-quality firm, with the cost decrease \( \Delta c_H \) from the \( c_H \) by process innovation, profit increment can be expressed as:
\[ \Delta \pi_{hi} = \pi_H \left( k_H, k_L, c_H - \Delta c_H, c_L \right) - \pi_H \left( k_H, k_L, c_H, c_L \right) \]

Obviously, the larger the \( \Delta c_H \), the stronger the process innovation ability of the firms.

Choices of Product and Process Innovation
The profit caused by process innovation is expressed with \( \Delta \pi_{hi} / \pi_{hi} \), and the profit by product innovation is \( \Delta \pi_{ih} / \pi_{ih} \). Making them compared, if \( \Delta \pi_{hi} / \pi_{hi} > \Delta \pi_{ih} / \pi_{ih} \), H firm selects process innovation; if \( \Delta \pi_{hi} / \pi_{hi} < \Delta \pi_{ih} / \pi_{ih} \), H firm selects product innovation; And if
\[ \frac{\Delta \pi_{hi}}{\pi_{hi}} = \frac{\Delta \pi_{ih}}{\pi_{ih}} \], H firm can select either process innovation or product innovation.

Take the high-quality firm as example to analyze the selection of innovation types both in the Bertrand and the Cournot competitions.
Firstly, assume that the product innovation is implemented, and the product quality is increased by $\Delta k_H$. Set the original value as $\pi_H^0$, $\bar{k}_H$, $\bar{c}_H$, $\bar{\alpha}_H$, $\bar{\tau}_H$. The $\Delta c_H^0$ can be calculated from $\Delta k_H$ with the equation (6). Put $\Delta k_H$, $\Delta c_H = \Delta c_H^0 \alpha_H$, and $\pi_H^0$, $\bar{k}_H$, $\bar{c}_H$, $\bar{\alpha}_H$, $\bar{\tau}_H$ into (1) and (3), arrange them, we can get:

$$\frac{\Delta \pi_H}{\pi_H} = N \left( \frac{\left( \mu_H + \Delta k_H \right)^2 - 2 \mu_H k_H - 2 \left( \mu_H - \Delta c_H \right) \mu_H + \bar{\alpha}_H \mu_H^2}{(4 \mu_H - \bar{\tau}_H)^2} - 1 \right)$$

$$\frac{\Delta \pi_H}{\pi_H} = N \left( \frac{\left( \mu_H + \Delta k_H \right)^2 - 2 \mu_H k_H - 2 \left( \mu_H - \Delta c_H \right) \mu_H + \bar{\alpha}_H \mu_H^2}{(4 \mu_H - \bar{\tau}_H)^2} - 1 \right)$$

$$\frac{\Delta \pi_H}{\pi_H} = N \left( \frac{\left( \mu_H + \Delta k_H \right)^2 - 2 \mu_H k_H - 2 \left( \mu_H - \Delta c_H \right) \mu_H + \bar{\alpha}_H \mu_H^2}{(4 \mu_H - \bar{\tau}_H)^2} - 1 \right)$$

Secondly, assume the H firm implements the process innovation, making the unit cost decreased by $\Delta c_H$. The profit increment is expressed as:

$$\frac{\Delta \pi_H}{\pi_H} = N \left( \frac{\left( \mu_H + \Delta k_H \right)^2 - 2 \mu_H k_H - 2 \left( \mu_H - \Delta c_H \right) \mu_H + \bar{\alpha}_H \mu_H^2}{(4 \mu_H - \bar{\tau}_H)^2} - 1 \right)$$

Let $\frac{\Delta \pi_H}{\pi_H} = \frac{\Delta \pi_H}{\pi_H}$, $\frac{\Delta \pi_H}{\pi_H} = \frac{\Delta \pi_H}{\pi_H}$, the relationship of $\frac{\Delta k_H}{k_H}$, $\frac{\Delta c_H}{c_H}$, $\alpha_H$ can be expressed as:

$$f_B \left( \frac{\Delta k_H}{k_H}, \frac{\Delta c_H}{c_H}, \alpha_H \right) = 0$$

$$f_C \left( \frac{\Delta k_H}{k_H}, \frac{\Delta c_H}{c_H}, \alpha_H \right) = 0$$
It shows that if the two equations are satisfied, the profits from both product innovation and process innovation are equal. When \( \frac{\Delta k_H}{k_H} > \frac{\Delta c_H}{c_H} \), it shows that for the same profits, more product innovation is needed, and the firm would select process innovation. On the contrary, when \( \frac{\Delta k_H}{k_H} < \frac{\Delta c_H}{c_H} \), for the same profits, more process innovation is needed, so the firm would select product innovation. We call this equation as criterion equation for innovation type. These equations can be illustrated as figure 1.

**Figure 1-- The Criterion Curves of Innovation Type for High-Quality Firm**

In the area above the diagonal of the figure, \( \frac{\Delta k_H}{k_H} < \frac{\Delta c_H}{c_H} \), the firm tends to select product innovation. Below the diagonal, \( \frac{\Delta k_H}{k_H} > \frac{\Delta c_H}{c_H} \), the firm would select process innovation.

With the same analyzing method, we can get the criterion equation of innovation type for the low-quality firm:
Figure 2-- The Criterion Curves of Innovation Type for Low-Quality Firm

From figure 1 and figure 2, we can see: When cost coefficient of product innovation $\alpha_{kH}$ or $\alpha_{kL}$ is small, that is when product innovation ability of the firms is strong, the criterion curves are in the upper area of the figure, and when $\alpha_{kH}$ or $\alpha_{kL}$ is large, the curves are in the lower area.

For the high-quality firm, at the same $\alpha_{kH}$, the criterion curve in Bertrand competition is upper than that in Cournot competition. That is to say in the Bertrand competition, the firm tend to select product innovation compared with Cournot competition. When $\alpha_{kH}$ is equal to zero, the firm has the largest product innovation ability, and would select product innovation in both the Bertrand and Cournot competition. When $\alpha_{kH}$ reaches to 0.12 in this example, the firm begin to complement process innovation in the Cournot competition, while the Bertrand competitor still selects product innovation. When the firm product innovation ability drops further with $\alpha_{kH}$ to be 0.2, the product
innovation begins to be implemented in the Bertrand competition. In this case, process innovation is carried out in both the Bertrand and Cournot competition.

For the low-quality firm, at the same $\alpha_{LL}$, the criterion curve in Cournot competition is above that in Bertrand competition. That is to say in the Cournot competition, the firm tend to select product innovation compared with Bertrand competition. When $\alpha_{HL}$ is equal to zero, the firm has the largest product innovation ability, and would select product innovation in both the Bertrand and Cournot competition. When $\alpha_{HL}$ reaches to 0.06 in this example, the firm begin to complement process innovation in the Bertrand competition, while the Cournot competitor still selects product innovation. When the firm product innovation ability decreases to 0.12, the product innovation begins to be implemented in the Cournot competition. In this case, process innovation is carried out in both the Bertrand and Cournot competition.

Conclusions

- When cost coefficient of product innovation is small, firms have large product innovation ability and have the bigger incentives and tendency of innovation, and vice versa.

- For the high-quality firms, at the same $\alpha_{HL}$, the firms tend to implement product innovation in the Bertrand competition. For the low-quality firms, at the same $\alpha_{LL}$, the firms tend to implement product innovation in the Cournot competition.

- When $\alpha_{m}$ or $\alpha_{L}$ equals to zero (the firms with the largest product innovation ability), both high-quality and low-quality firms select product innovation in both Bertrand competition and Cournot competition. For the high-quality firms, when $\alpha_{HL}$ increases to a certain value, the process innovation would be selected in Cournot competition, while the product innovation is selected in the Bertrand competition. With the $\alpha_{HL}$ increased further, the process innovation would be selected in both Bertrand competition and Cournot competition. For the low-quality firms, when $\alpha_{LL}$ increases to a certain value,
the process innovation would be selected in Bertrand competition, while the product innovation is selected in Cournot competition. With the $\alpha_k$ increased further, the process innovation would be selected in both Bertrand competition and Cournot competition.

- The study in this paper is mainly limited within the case that firms in the duopoly market and have the vertical product differentiation, and firms decide their strategies at the same time. Product innovation ability is just reflected with the increase of the unit cost, and the process innovation ability is just reflected with the decrease of the cost. In fact, the differentiation including the vertical and horizontal ones, and the innovation ability has many factors to reflect. How to reflect the factors which is near the reality as close as possible in the model is the subject to pursue in the future research.

References


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